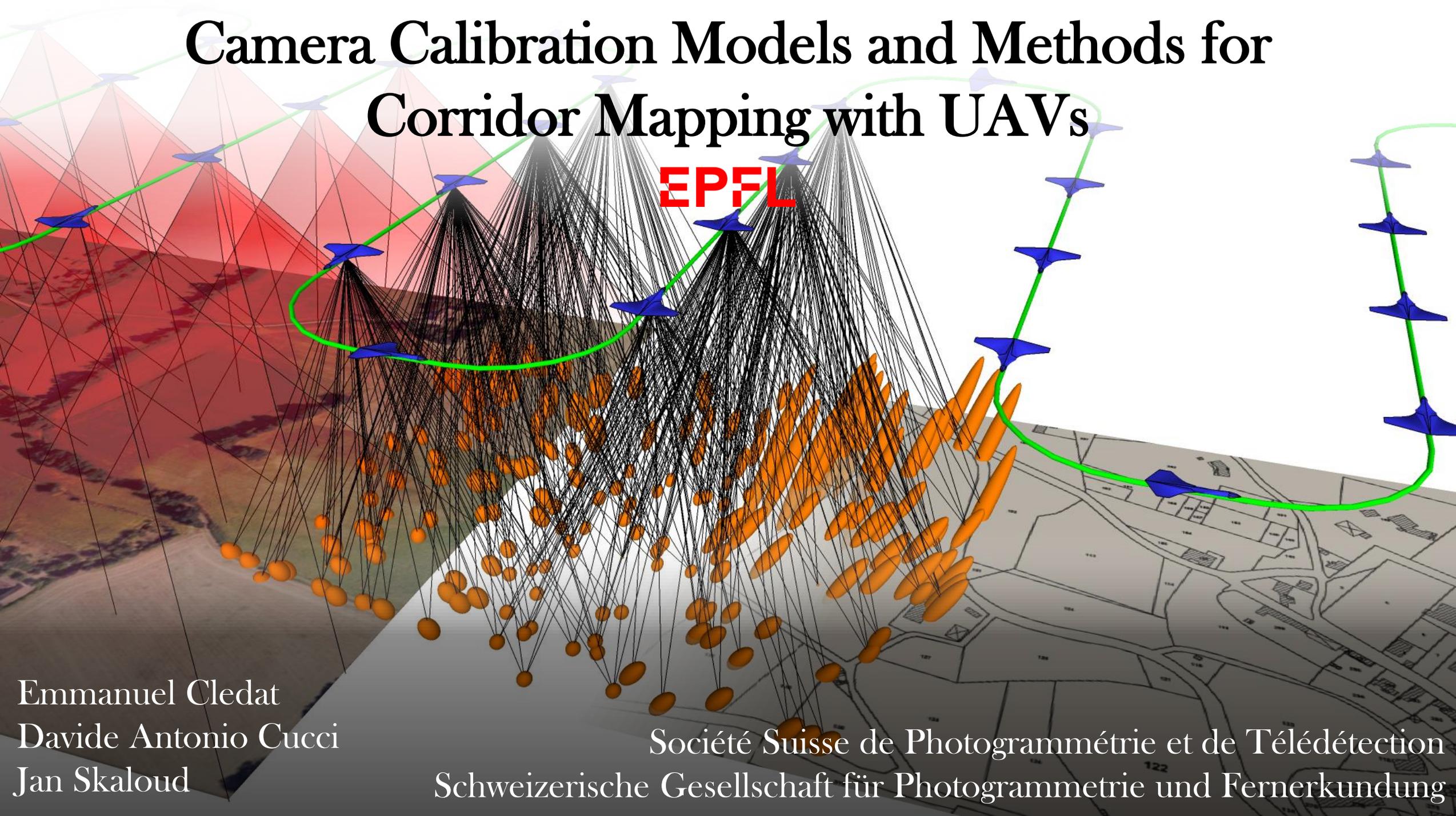


# Camera Calibration Models and Methods for Corridor Mapping with UAVs

EPFL

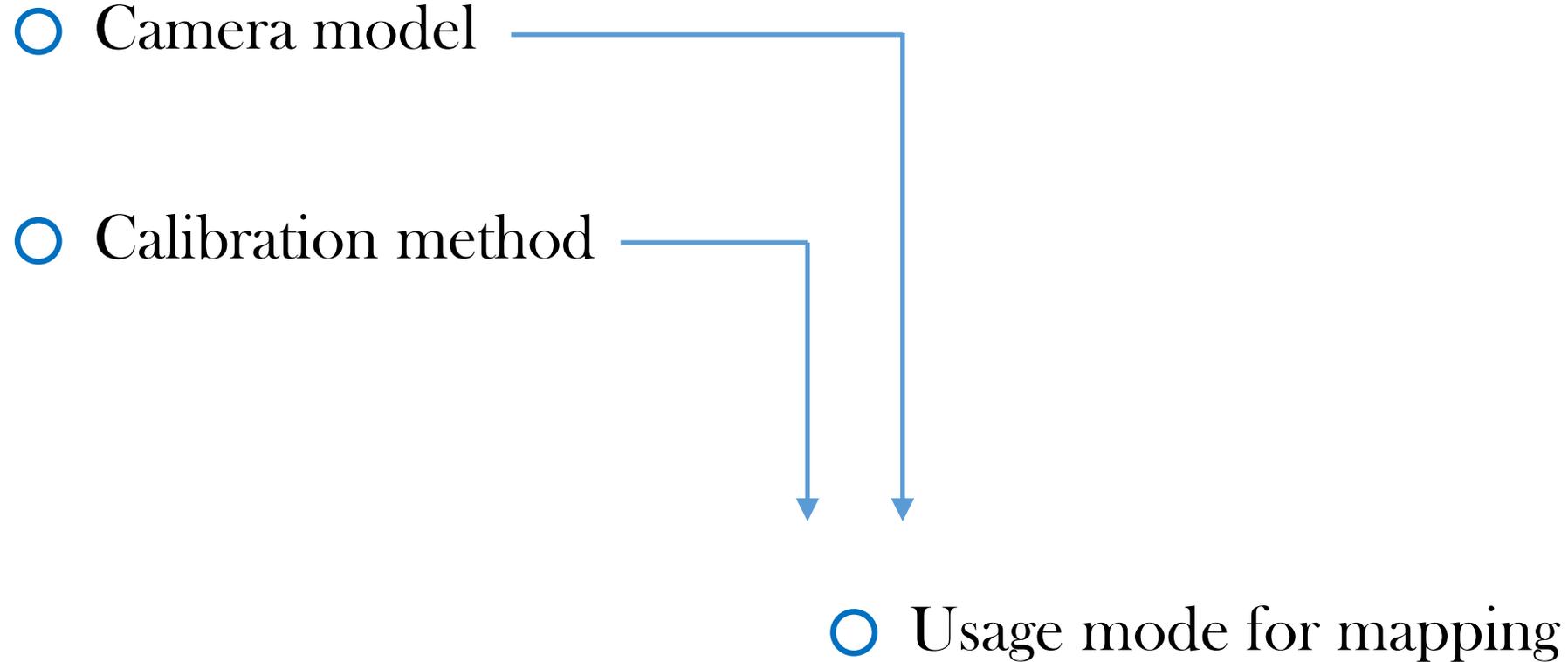
Emmanuel Cledat  
Davide Antonio Cucci  
Jan Skaloud

Société Suisse de Photogrammétrie et de Télédétection  
Schweizerische Gesellschaft für Photogrammetrie und Fernerkundung



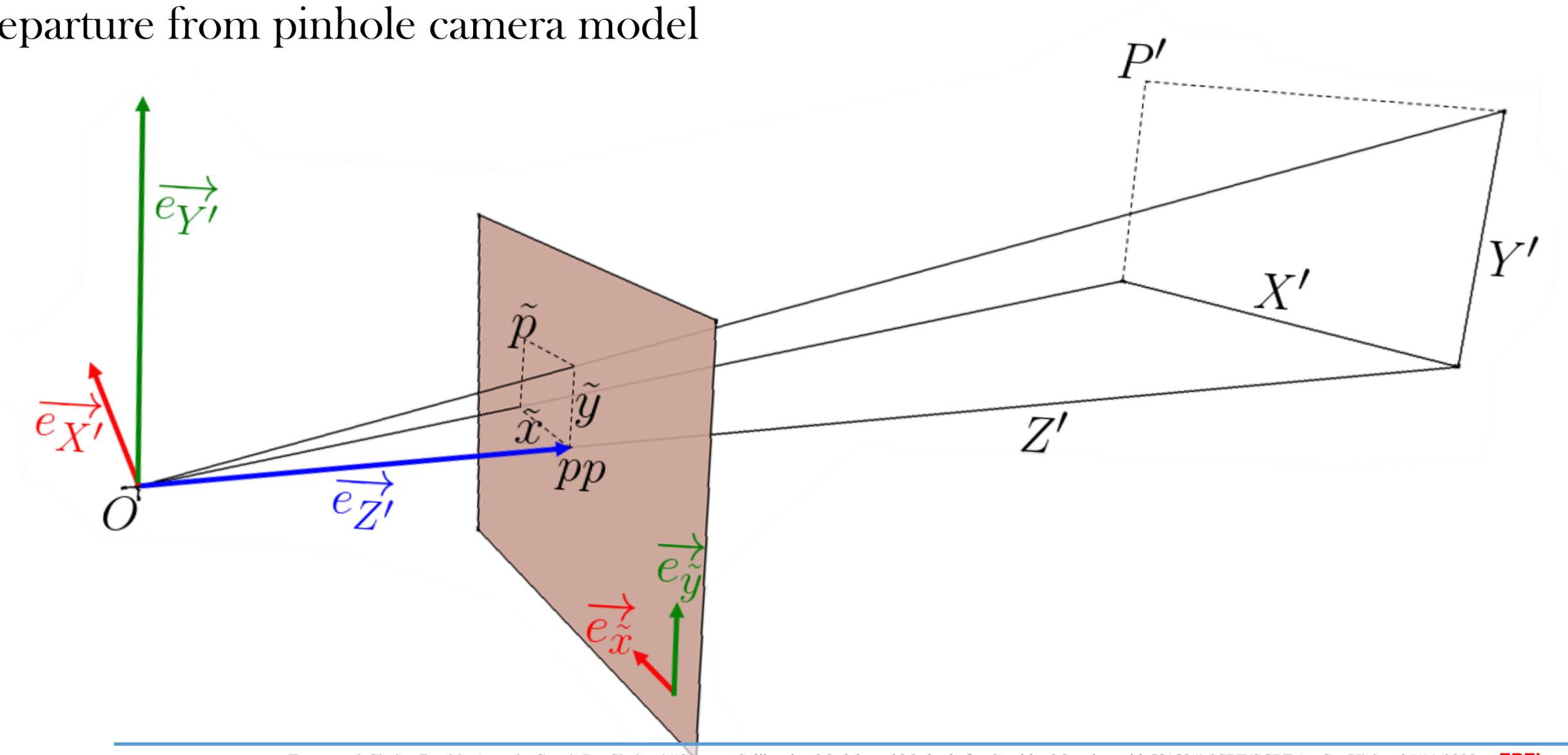
# Camera Calibration Models and Methods... Why?

---

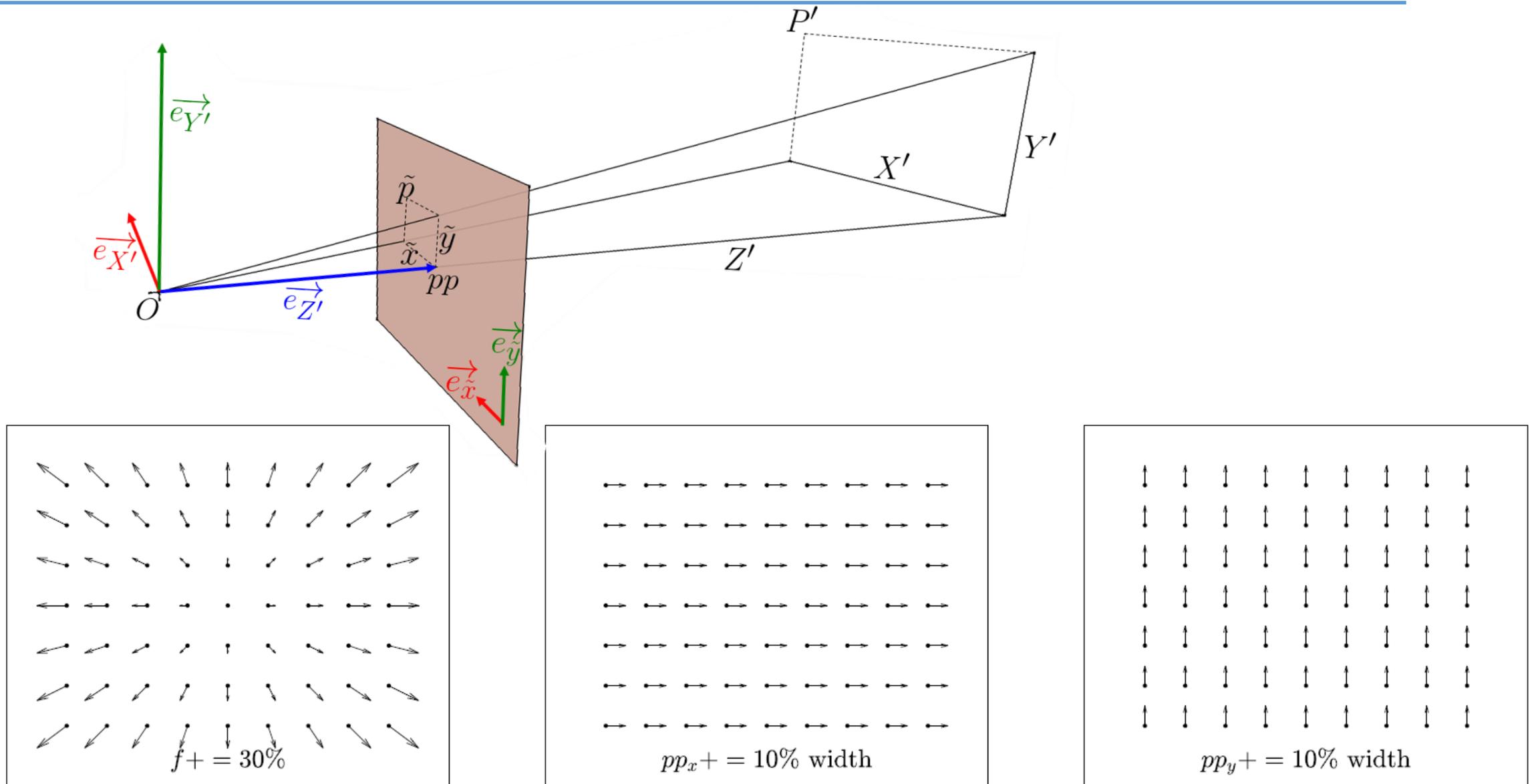


# Camera models...

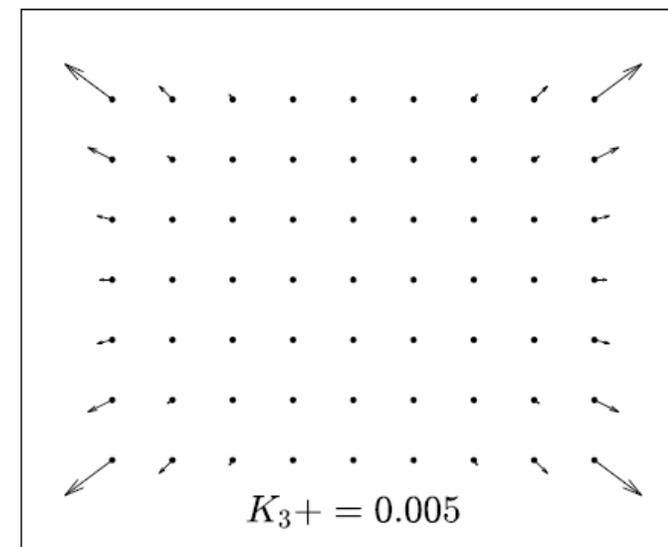
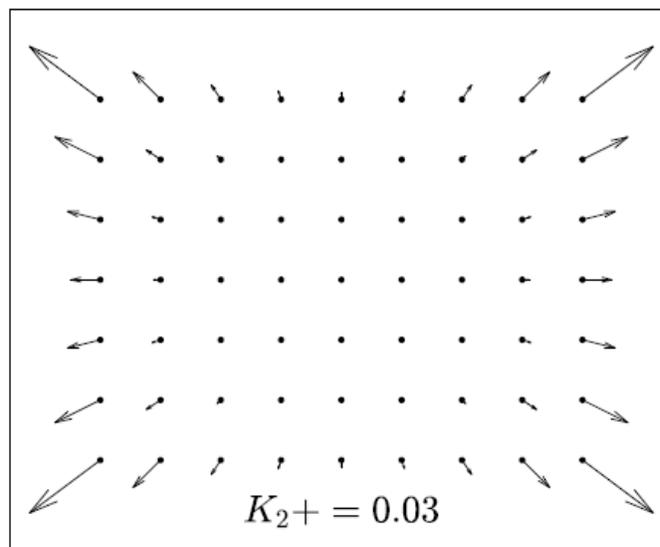
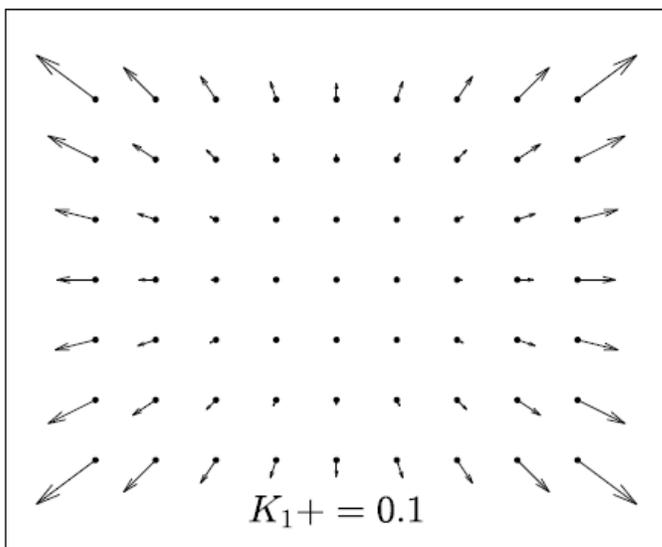
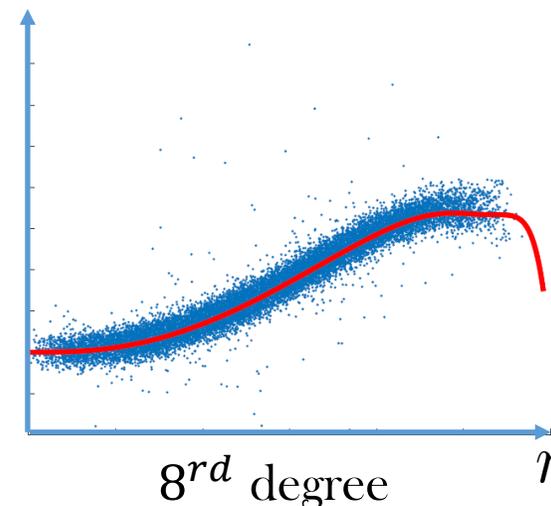
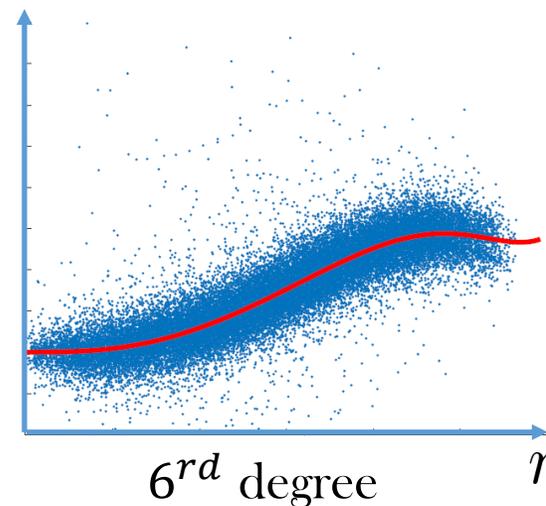
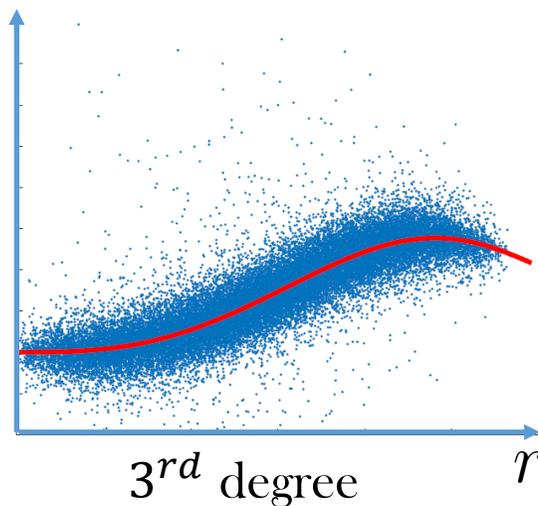
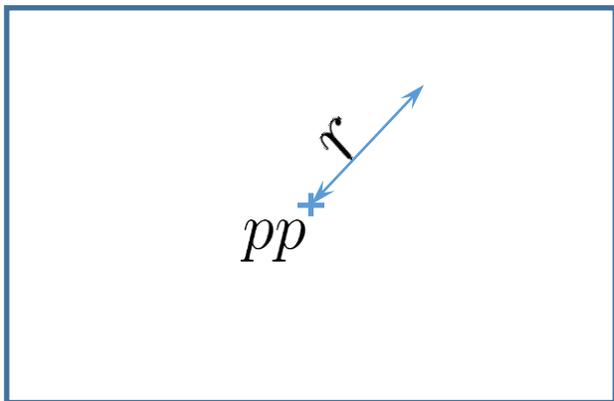
## Departure from pinhole camera model



# Camera models: Principal point position & Principal distance



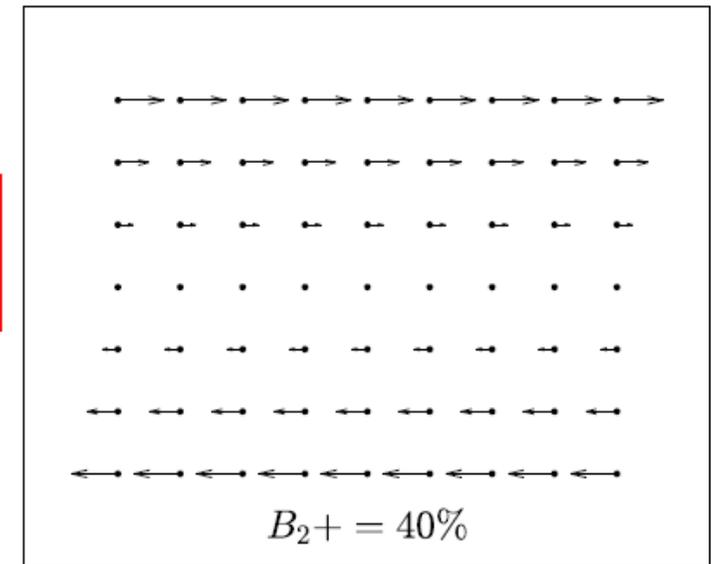
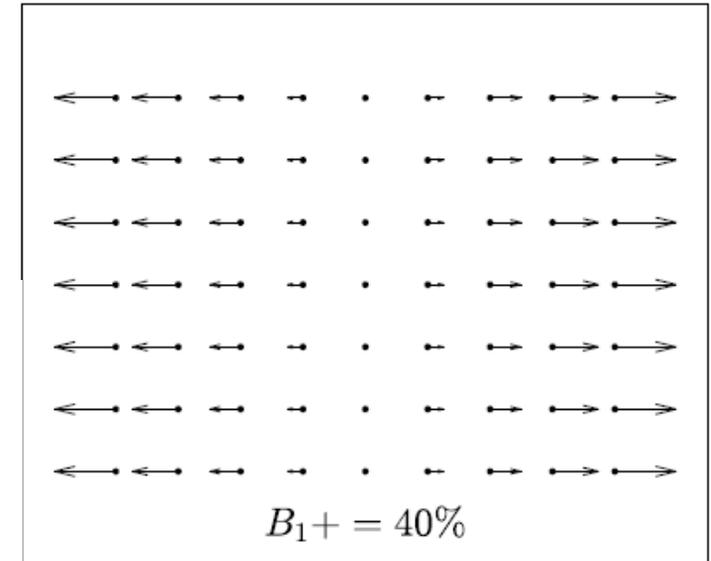
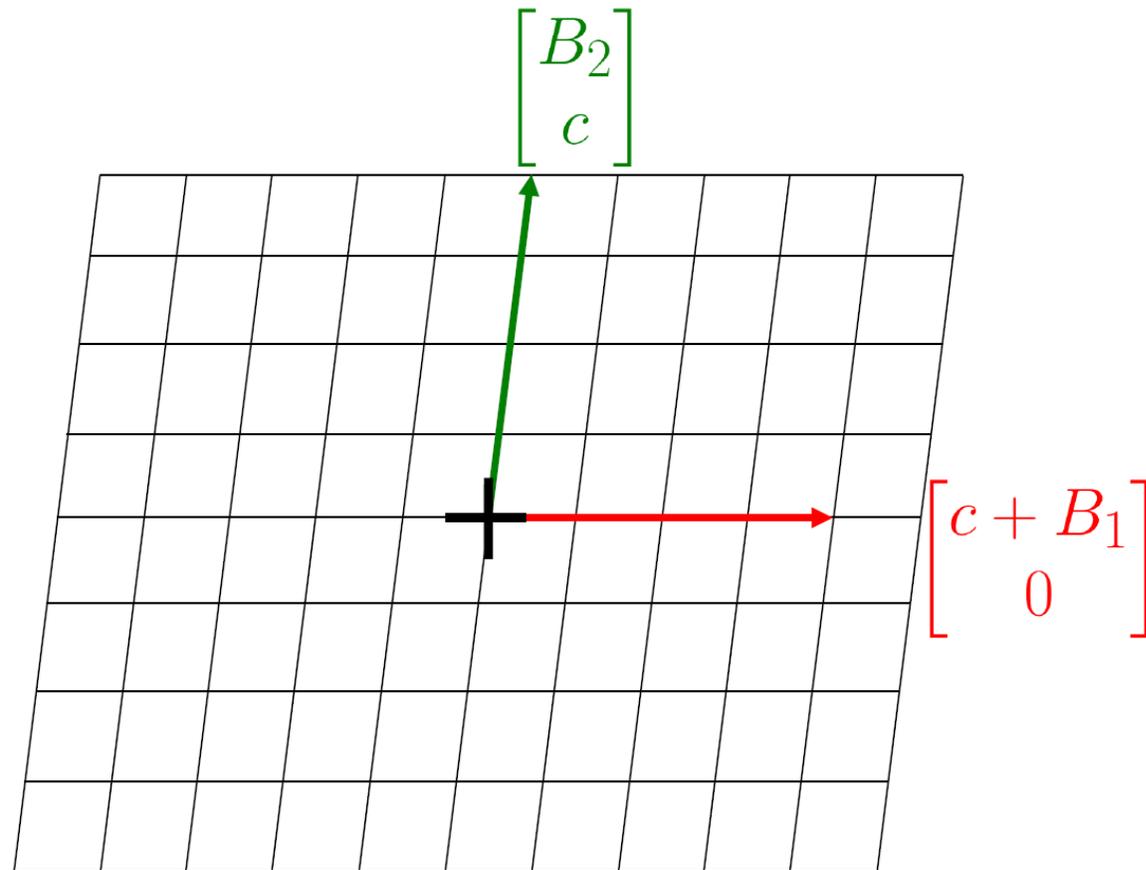
# Camera models: Radials distortions



# Camera models: Skew parameters

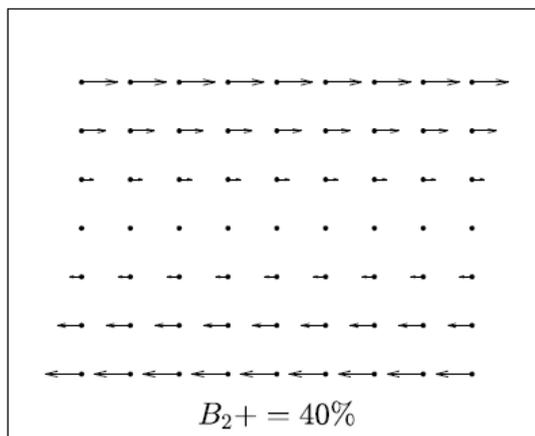
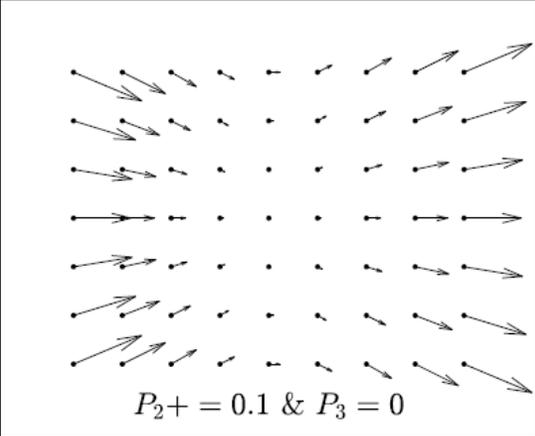
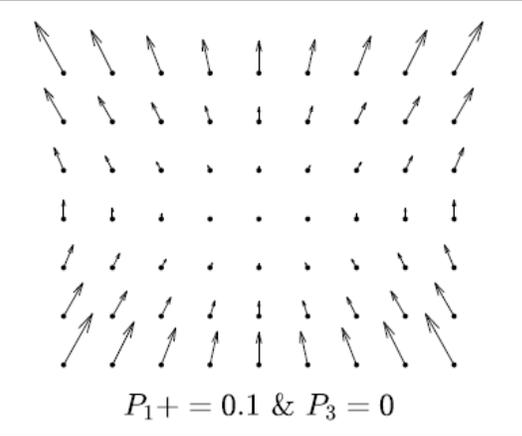
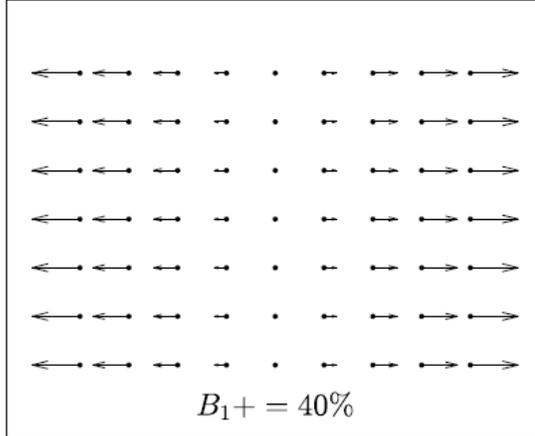
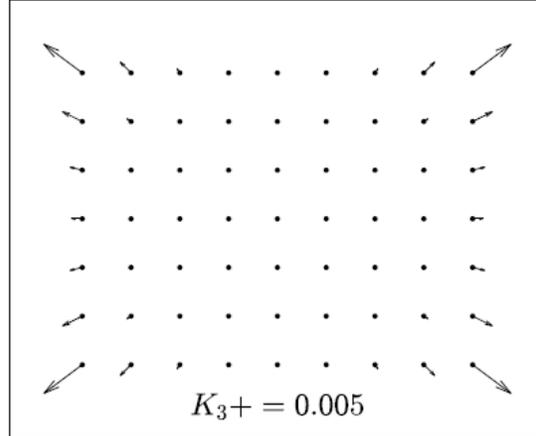
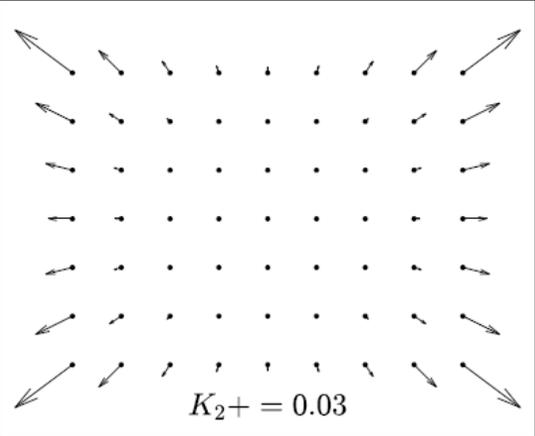
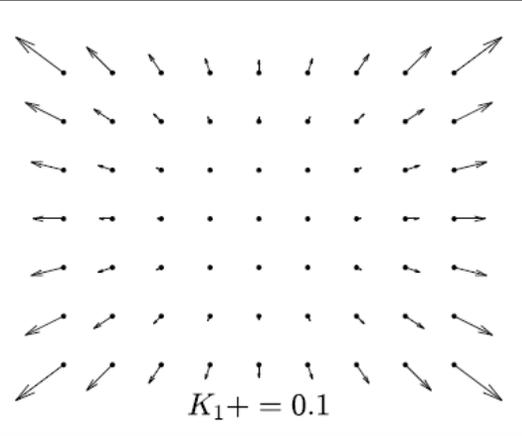
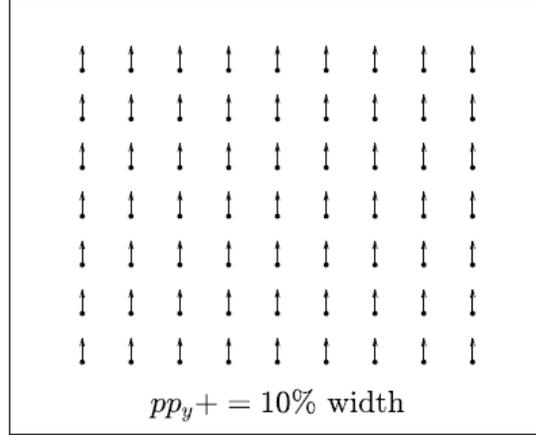
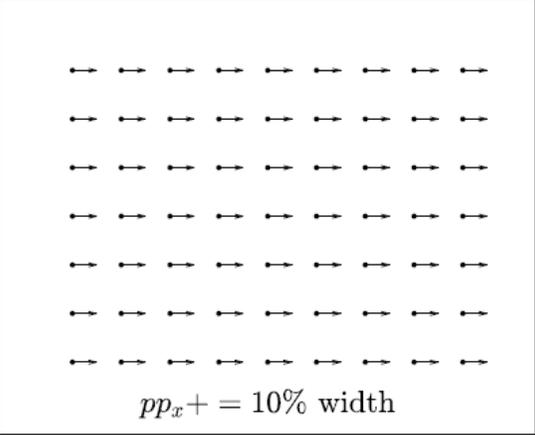
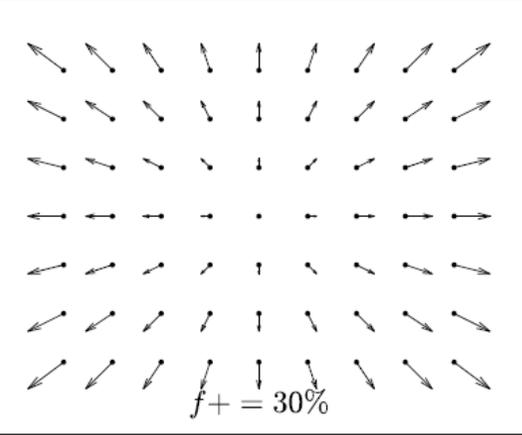
$$\xi_2: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} c + B_1 & B_2 \\ 0 & c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} pp_x \\ pp_y \end{bmatrix}$$



# Brown

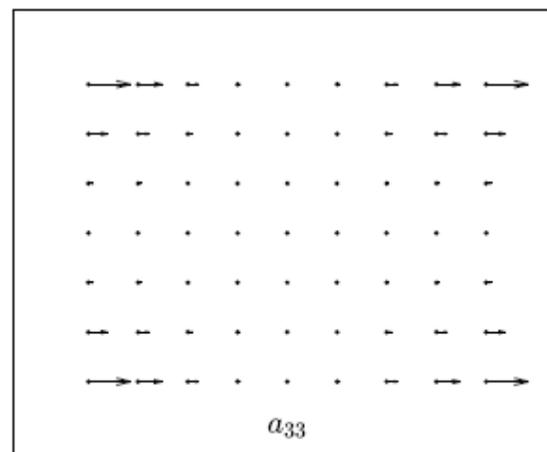
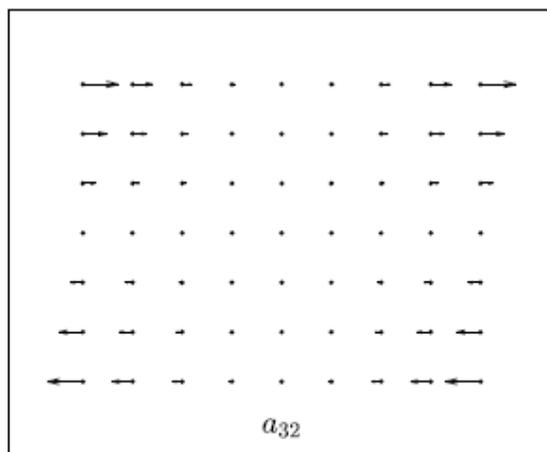
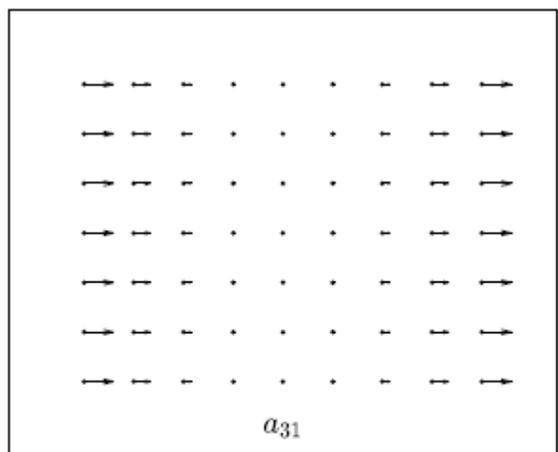
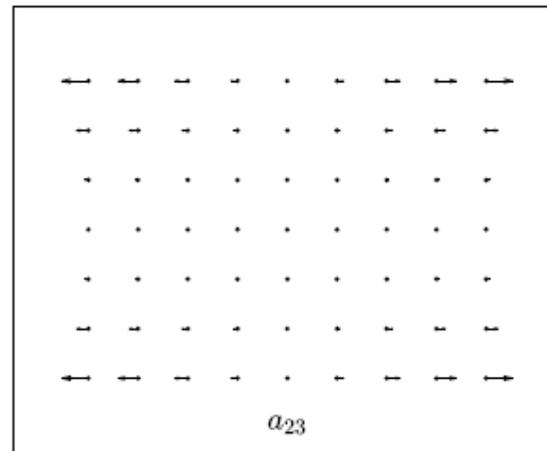
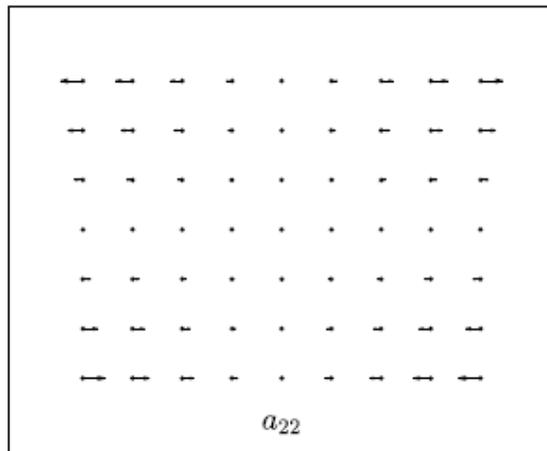
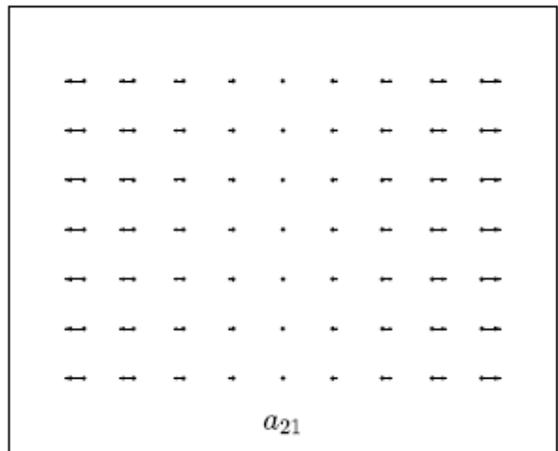
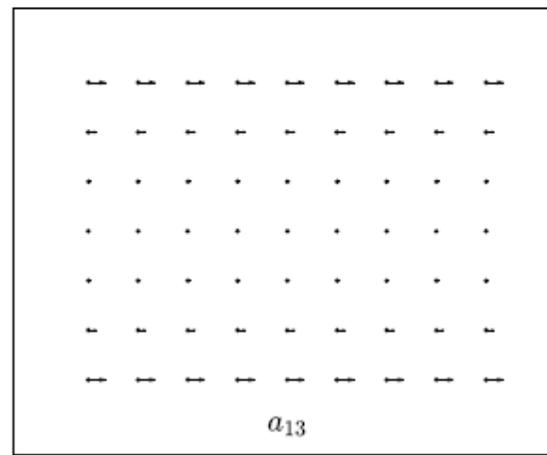
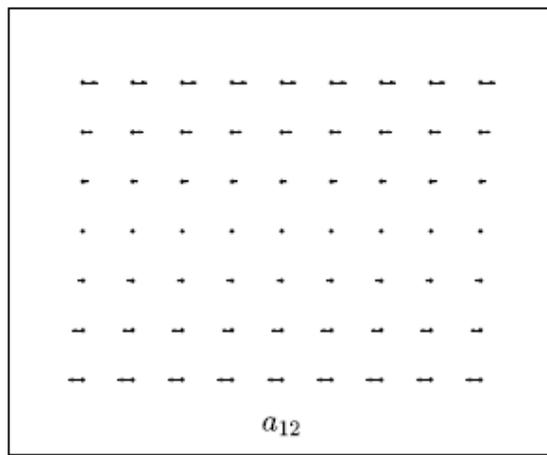
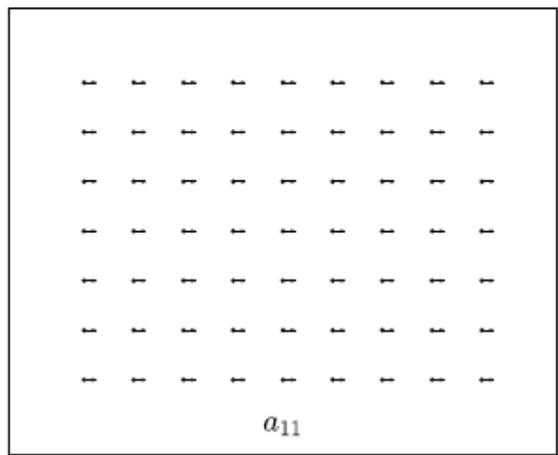
10 parameters  
version



# Orthogonal Polynomials

18 parameters

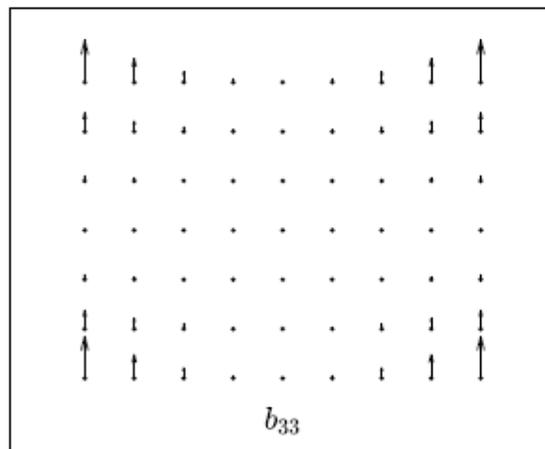
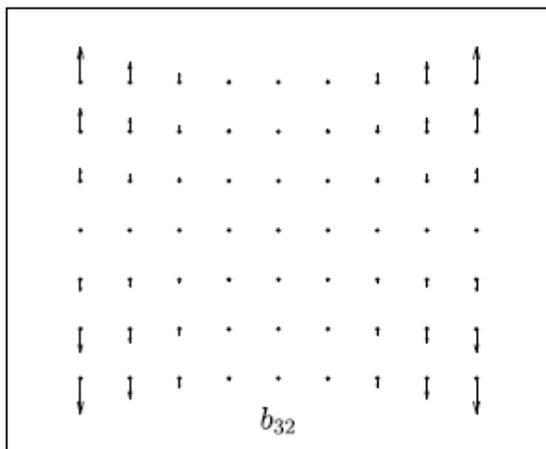
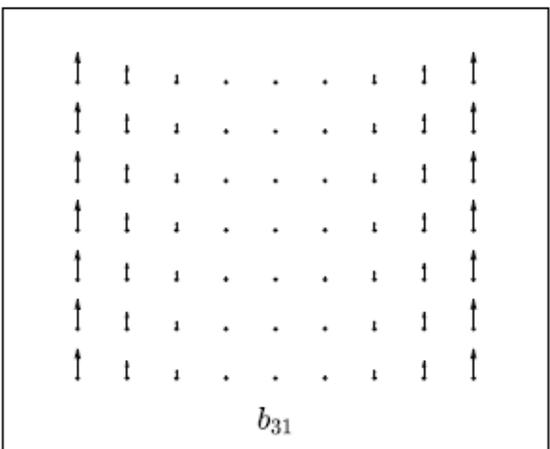
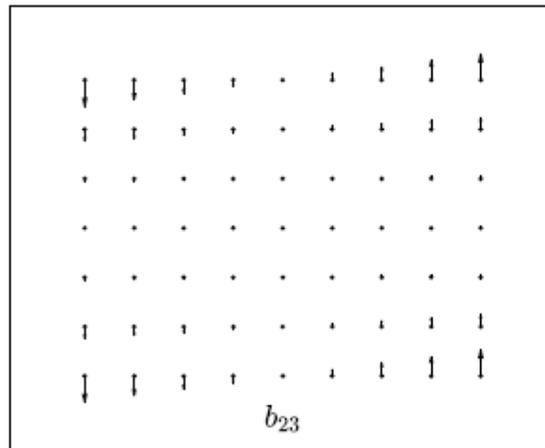
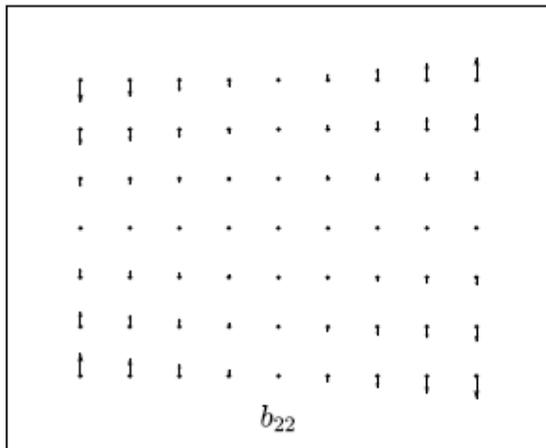
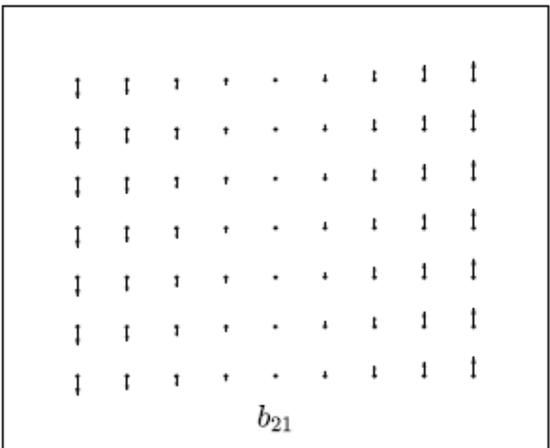
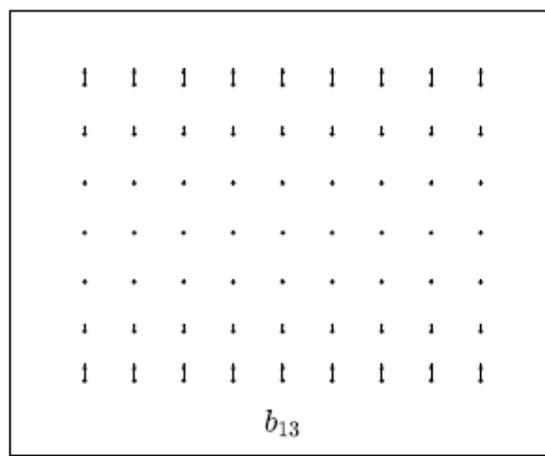
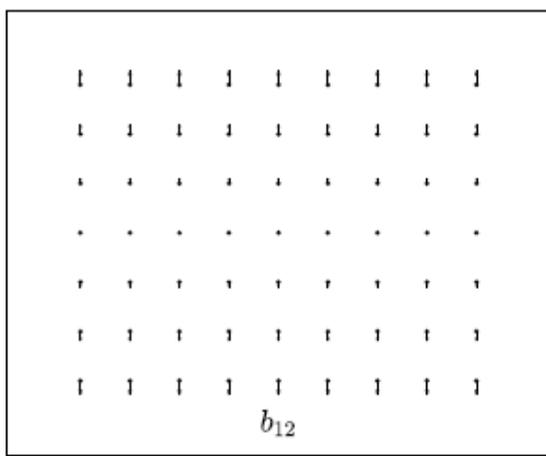
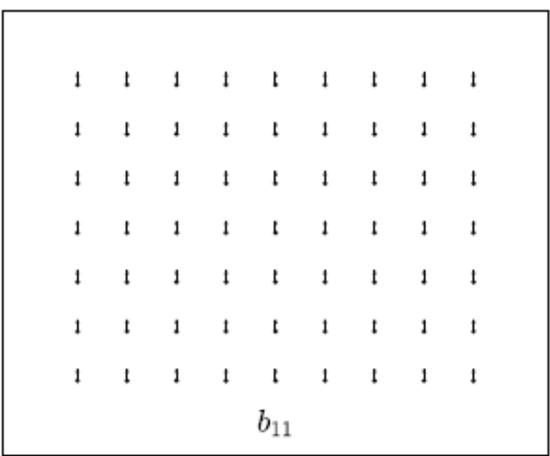
$$\left\{ \begin{array}{l} \ell_x = \begin{bmatrix} 1 \\ \tilde{x} \\ \tilde{x}^2 - \frac{2}{3}b_x^2 \end{bmatrix}^T \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 1 \\ \tilde{y} \\ \tilde{y}^2 - \frac{2}{3}b_y^2 \end{bmatrix} \\ \ell_y = \begin{bmatrix} 1 \\ \tilde{x} \\ \tilde{x}^2 - \frac{2}{3}b_x^2 \end{bmatrix}^T \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} 1 \\ \tilde{y} \\ \tilde{y}^2 - \frac{2}{3}b_y^2 \end{bmatrix} \end{array} \right.$$



# Orthogonal Polynomials

18 parameters

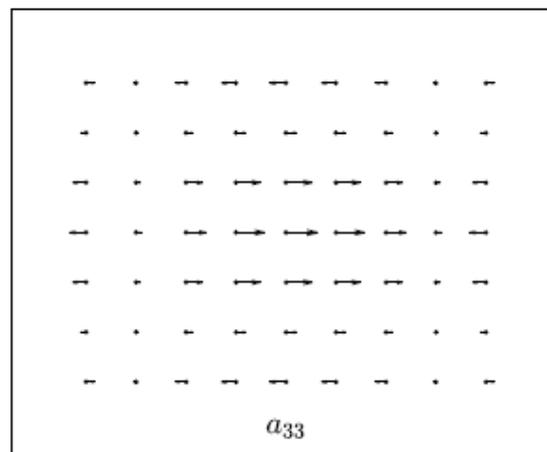
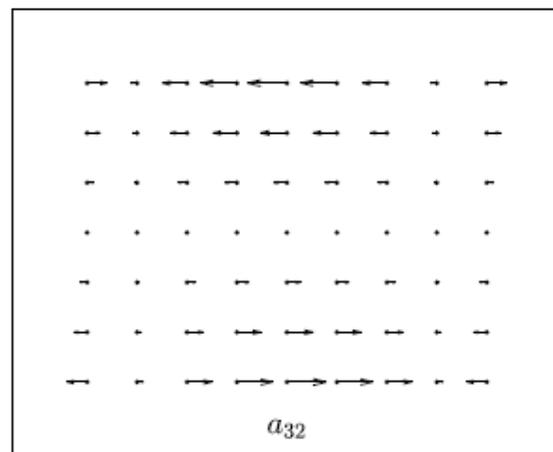
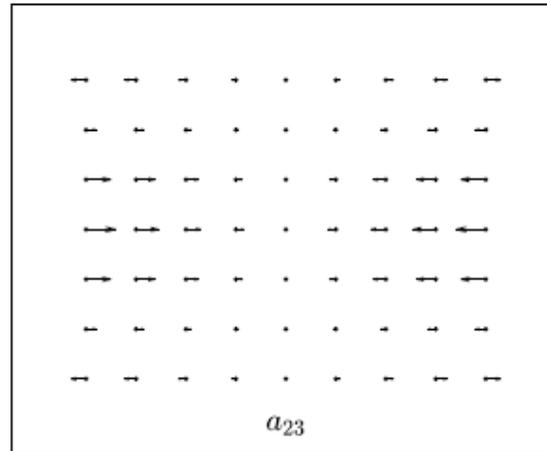
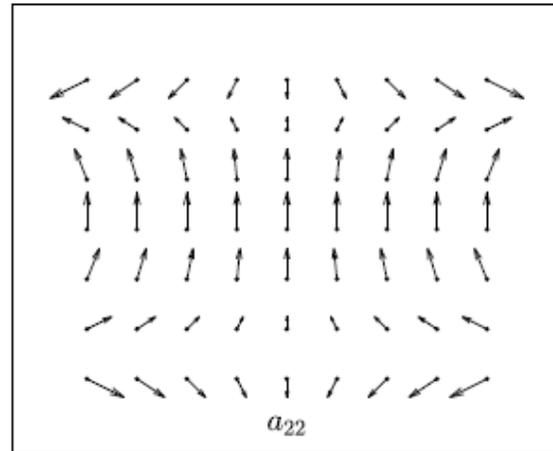
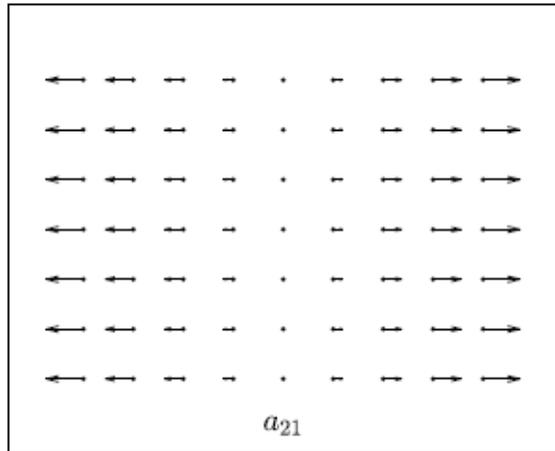
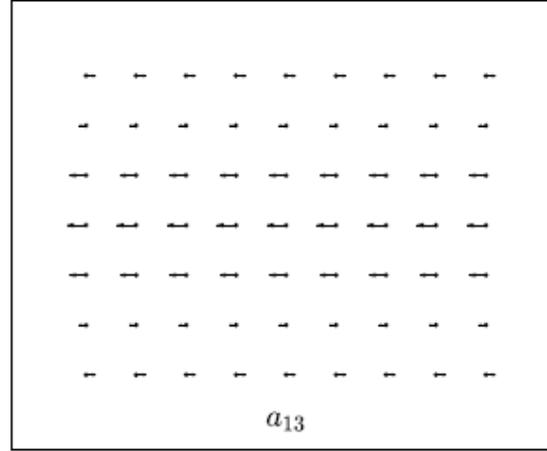
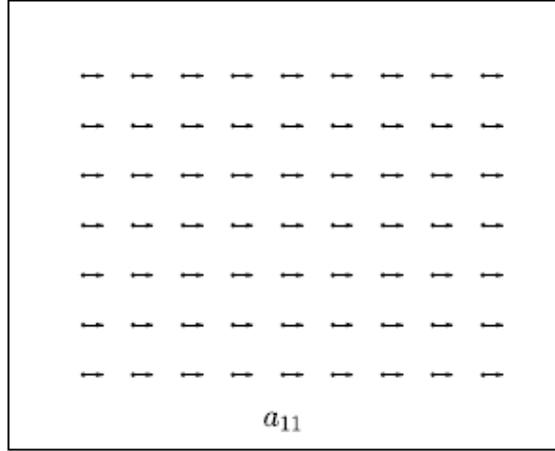
$$\left\{ \begin{array}{l} \ell_x = \begin{bmatrix} 1 \\ \tilde{x} \\ \tilde{x}^2 - \frac{2}{3}b_x^2 \end{bmatrix}^T \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 1 \\ \tilde{y} \\ \tilde{y}^2 - \frac{2}{3}b_y^2 \end{bmatrix} \\ \ell_y = \begin{bmatrix} 1 \\ \tilde{x} \\ \tilde{x}^2 - \frac{2}{3}b_x^2 \end{bmatrix}^T \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} 1 \\ \tilde{y} \\ \tilde{y}^2 - \frac{2}{3}b_y^2 \end{bmatrix} \end{array} \right.$$



# Orthogonal Polynomials

With 3 constraints

$$18 - 3 = 15$$



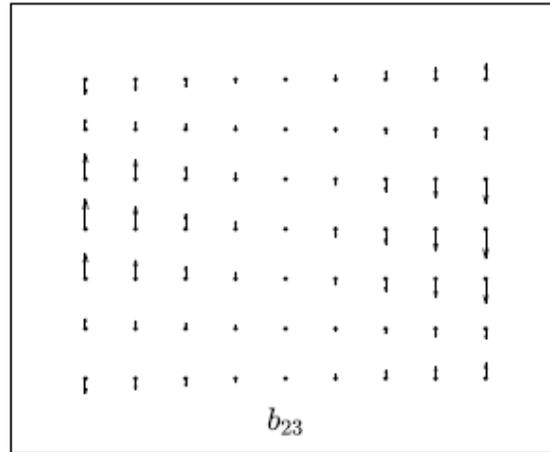
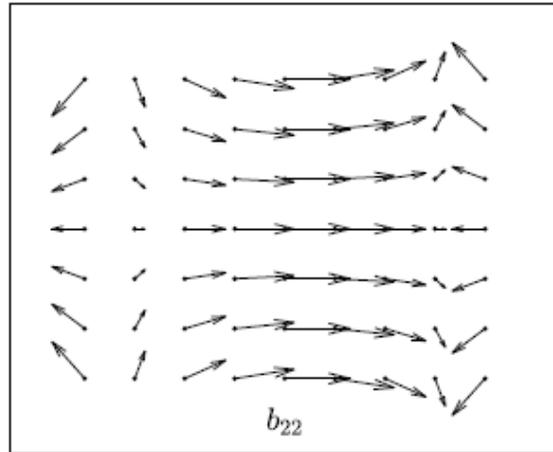
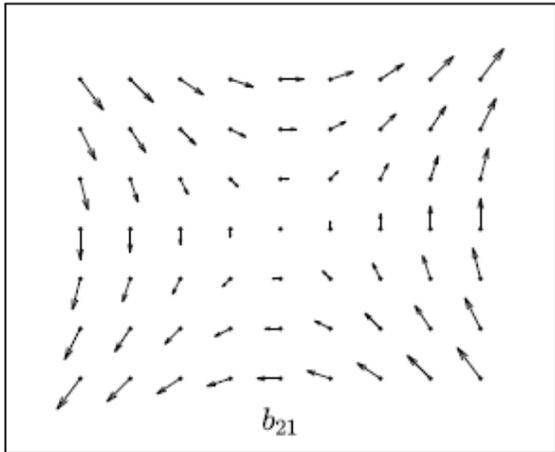
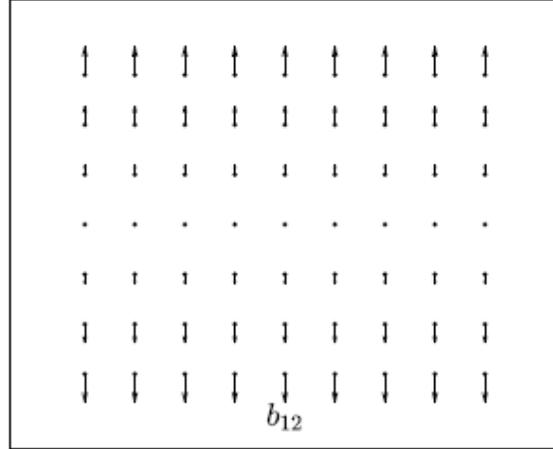
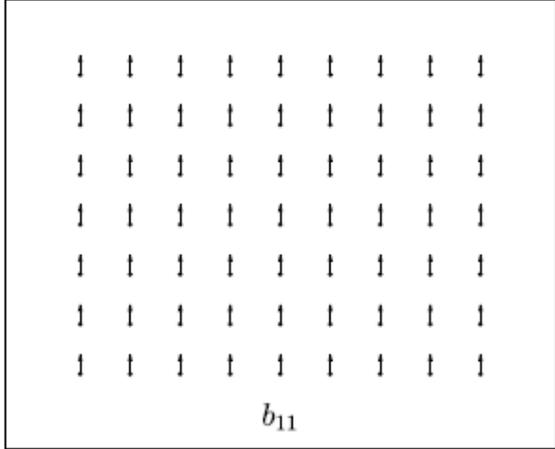
$$\left\{ \begin{array}{l} \ell_x = \begin{bmatrix} 1 \\ \tilde{x} \\ \tilde{x}^2 - \frac{2}{3}b_x^2 \end{bmatrix}^T \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 1 \\ \tilde{y} \\ \tilde{y}^2 - \frac{2}{3}b_y^2 \end{bmatrix} \\ \ell_y = \begin{bmatrix} 1 \\ \tilde{x} \\ \tilde{x}^2 - \frac{2}{3}b_x^2 \end{bmatrix}^T \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} 1 \\ \tilde{y} \\ \tilde{y}^2 - \frac{2}{3}b_y^2 \end{bmatrix} \end{array} \right.$$

$$\begin{cases} b_{13} + 2 \cdot a_{22} = 0 \\ a_{31} + 2 \cdot b_{22} = 0 \\ a_{12} - b_{21} = 0 \end{cases}$$

# Orthogonal Polynomials

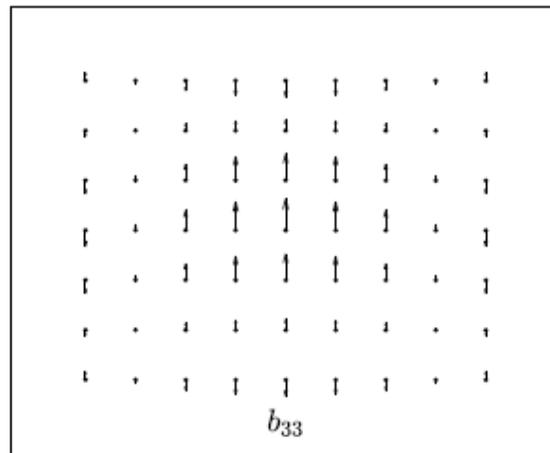
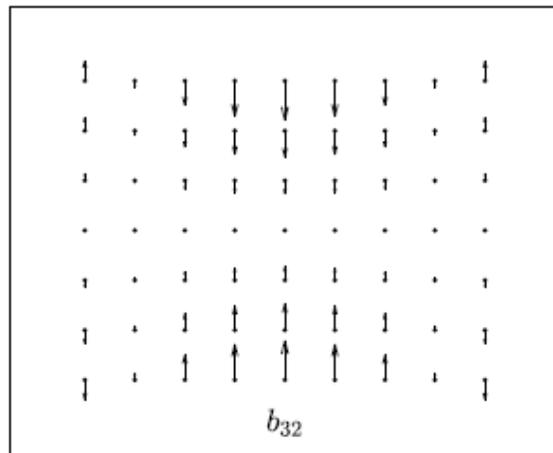
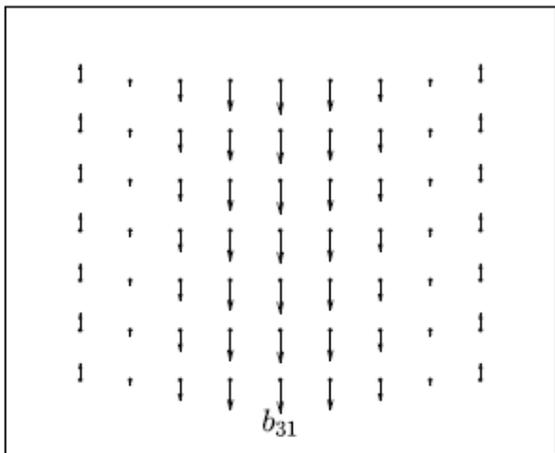
With 3 constraints

$$18 - 3 = 15$$



$$\ell_x = \begin{bmatrix} 1 \\ \tilde{x} \\ \tilde{x}^2 - \frac{2}{3}b_x^2 \end{bmatrix}^T \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 1 \\ \tilde{y} \\ \tilde{y}^2 - \frac{2}{3}b_y^2 \end{bmatrix}$$

$$\ell_y = \begin{bmatrix} 1 \\ \tilde{x} \\ \tilde{x}^2 - \frac{2}{3}b_x^2 \end{bmatrix}^T \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} 1 \\ \tilde{y} \\ \tilde{y}^2 - \frac{2}{3}b_y^2 \end{bmatrix}$$



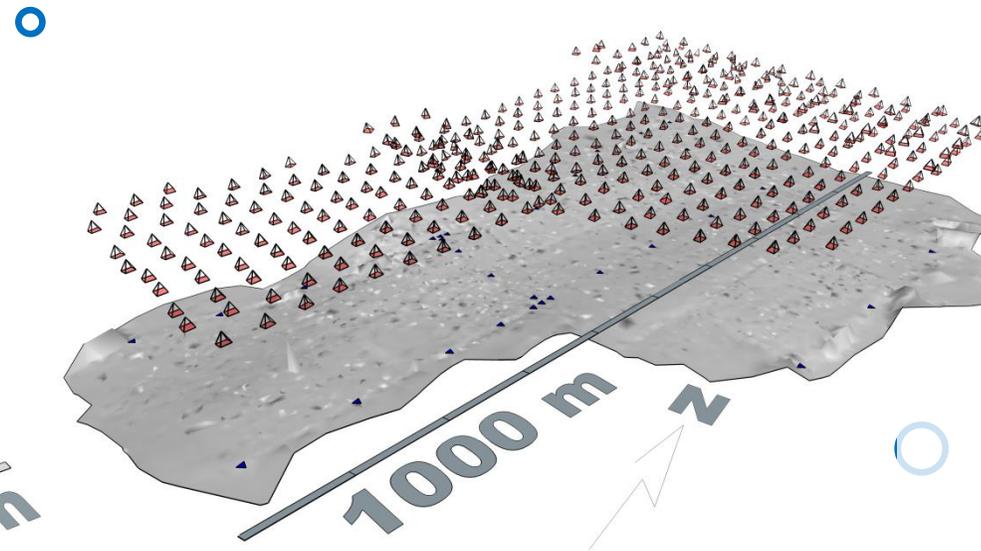
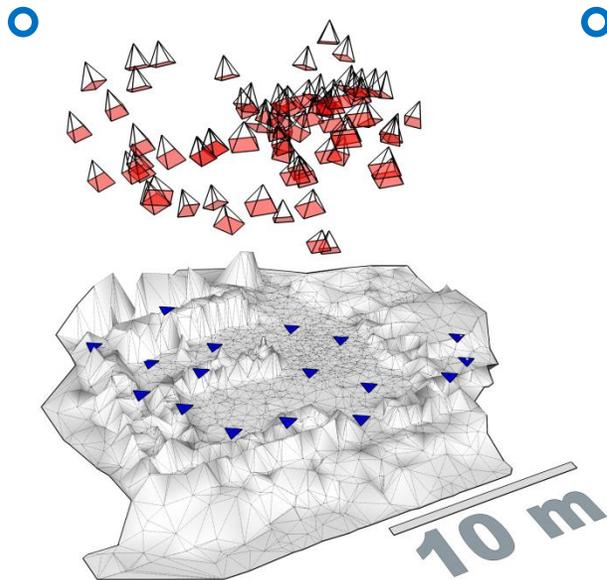
$$\begin{cases} b_{13} + 2 \cdot a_{22} = 0 \\ a_{31} + 2 \cdot b_{22} = 0 \\ a_{12} - b_{21} = 0 \end{cases}$$

# Camera calibration model and method: evaluation in application

## ○ Camera model

- Brown 10
- Brown 15
- Brown 18
- Ortho-Poly 18
- Ortho-Poly 15

## ○ Calibration method



## ○ Usage mode for mapping

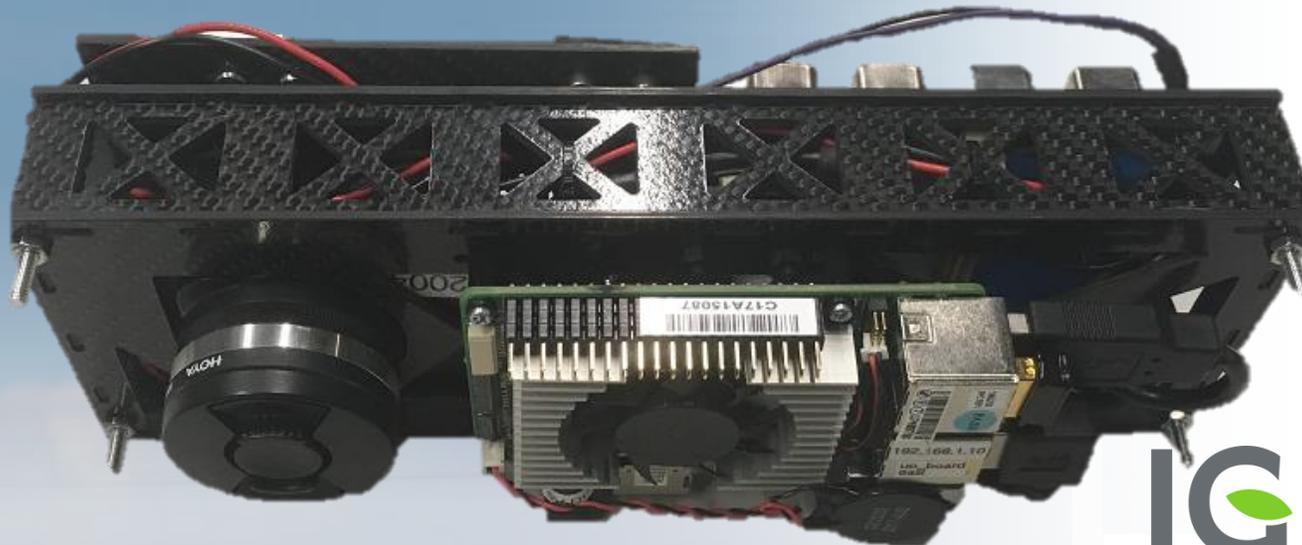
- FIX
- LEAD
- APC
- APCI

# CAM LIGHT from IGN [The French SwissTopo :-]

Fix-Wing Drone  
With aerial control



Hexacopter  
Without  
aerial  
control

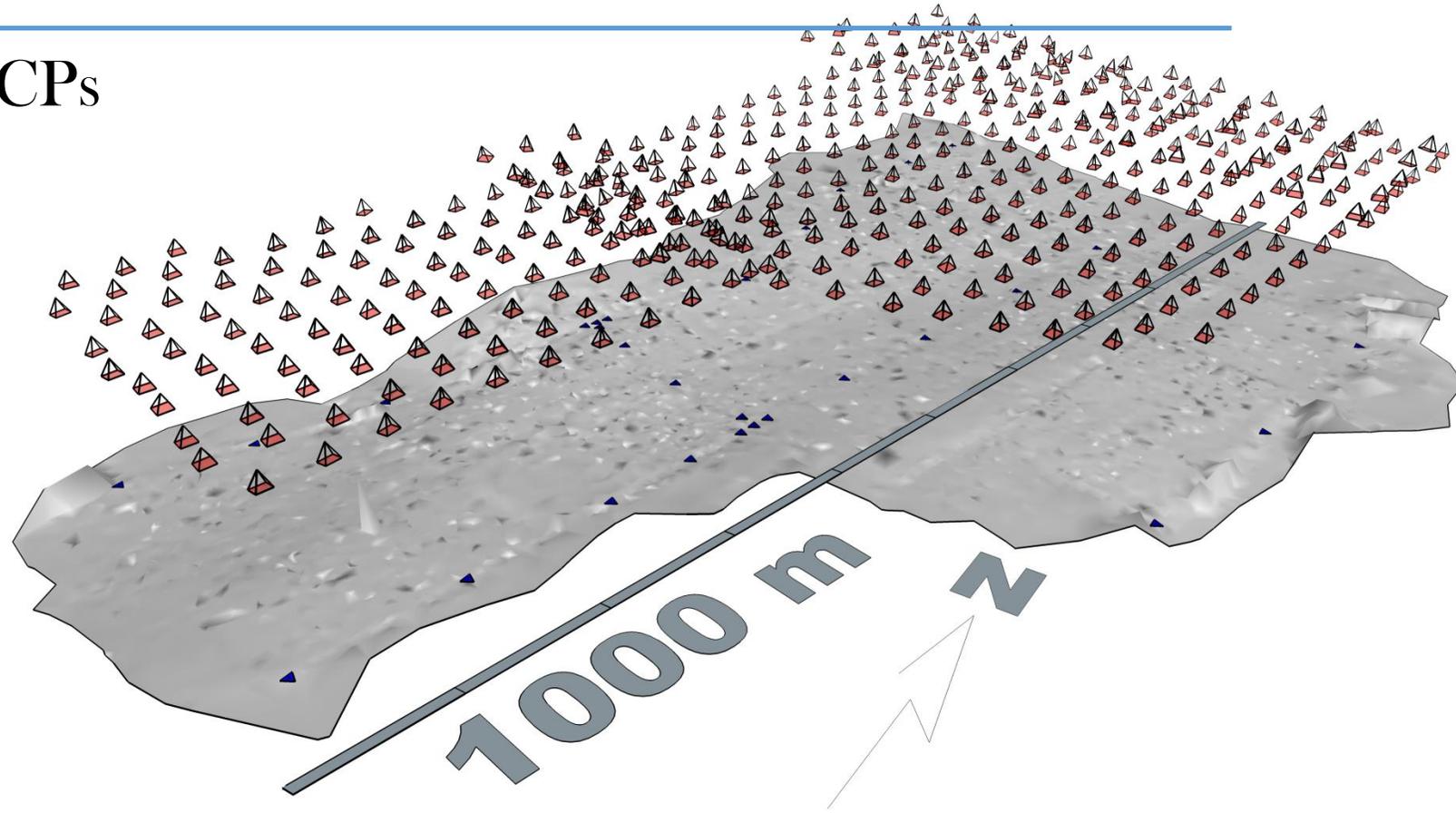
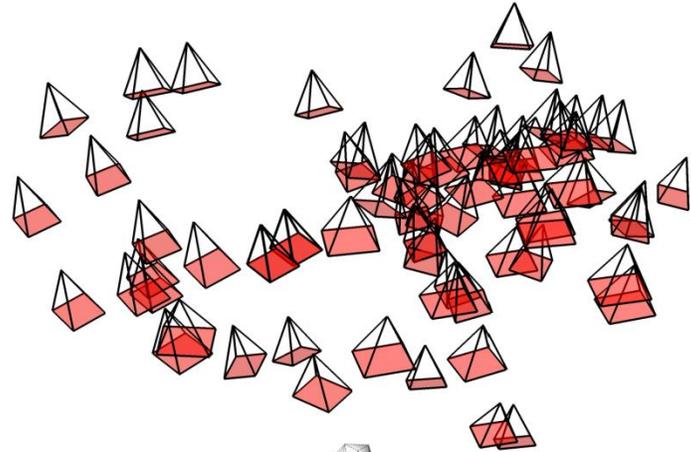


**IGN**  
INSTITUT NATIONAL  
DE L'INFORMATION  
GÉOGRAPHIQUE  
ET FORESTIÈRE

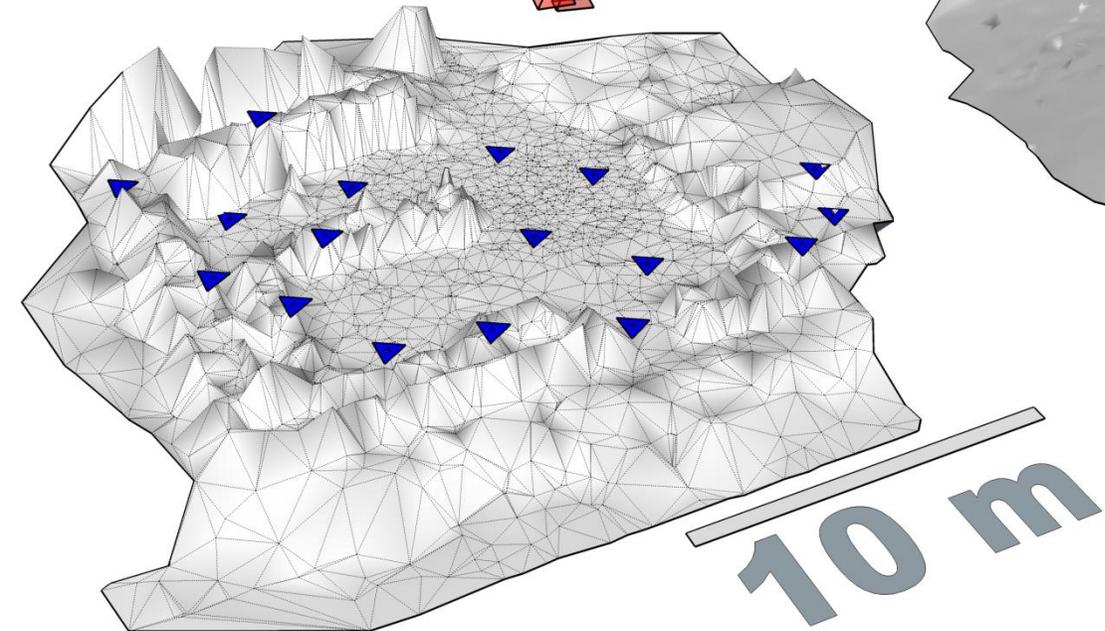


# Calibration field

Without aerial control, 17 GCPs



With aerial control  
21 GCPs



# Camera calibration model and method evaluation: SOTA

○ Images residuals

○ GCP residuals

○ CP misclosure

Of the  
calibration  
flight

Calibration  
parameters  
analysis

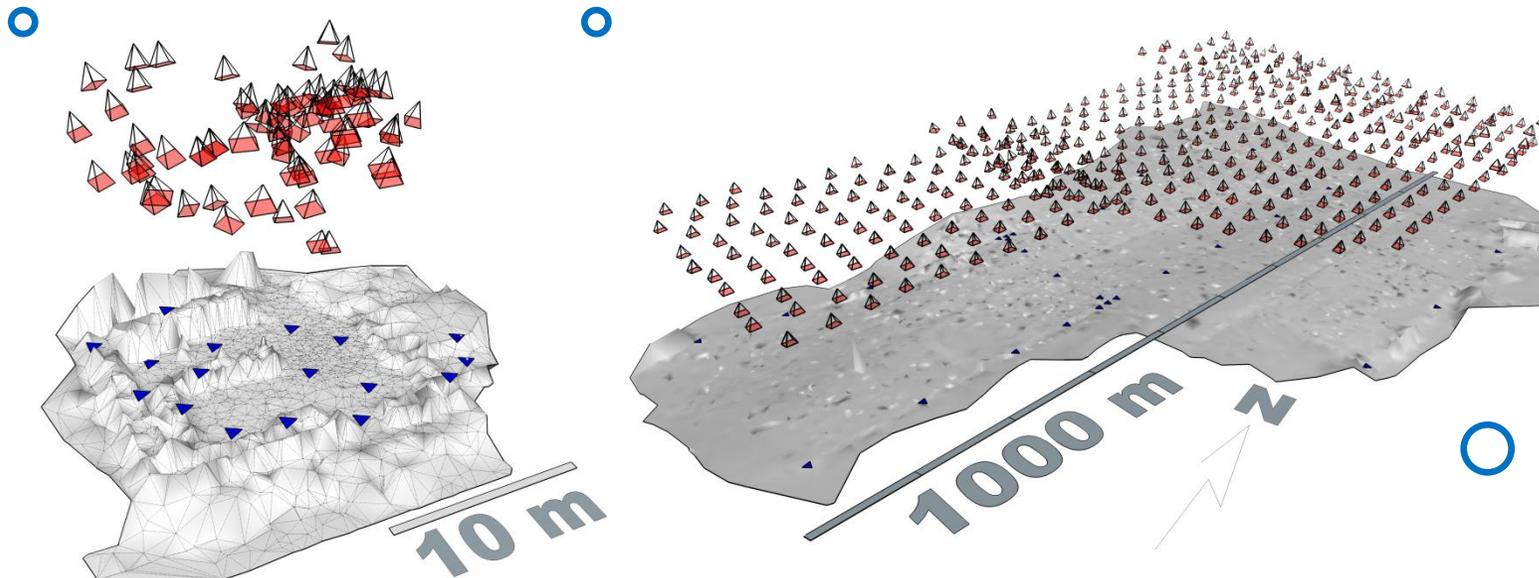
		f	ppx	ppy	R1	R2	R3	T1	T2	B1	B2
1.26	1.00	-0.28	-0.97	0.28	-0.02	0.01	-0.65	-0.12	0.73	-0.20	
1.46e-01	-0.28	1.00	0.28	-0.11	0.03	-0.03	0.21	0.87	-0.22	0.52	
6.56e-01	-0.97	0.28	1.00	-0.29	0.04	-0.03	0.68	0.11	-0.81	0.23	
1.88e-04	0.28	-0.11	-0.29	1.00	-0.90	0.85	-0.41	-0.08	0.07	-0.02	
1.32e-03	-0.02	0.03	0.04	-0.90	1.00	-0.99	0.07	0.01	-0.02	0.01	
3.03e-03	0.01	-0.03	-0.03	0.85	-0.99	1.00	-0.06	-0.01	0.01	-0.01	
1.78e-05	-0.65	0.21	0.68	-0.41	0.07	-0.06	1.00	0.15	-0.18	0.05	
7.69e-06	-0.12	0.87	0.11	-0.08	0.01	-0.01	0.15	1.00	-0.03	0.22	
6.71e-02	0.73	-0.22	-0.81	0.07	-0.02	0.01	-0.18	-0.03	1.00	-0.29	
2.26e-02	-0.20	0.52	0.23	-0.02	0.01	-0.01	0.05	0.22	-0.29	1.00	

# Camera calibration model and method: evaluation in application

## ○ Camera model

- Brown 10
- Brown 15
- Brown 18
- Ortho-Poly 18
- Ortho-Poly 15

## ○ Calibration method

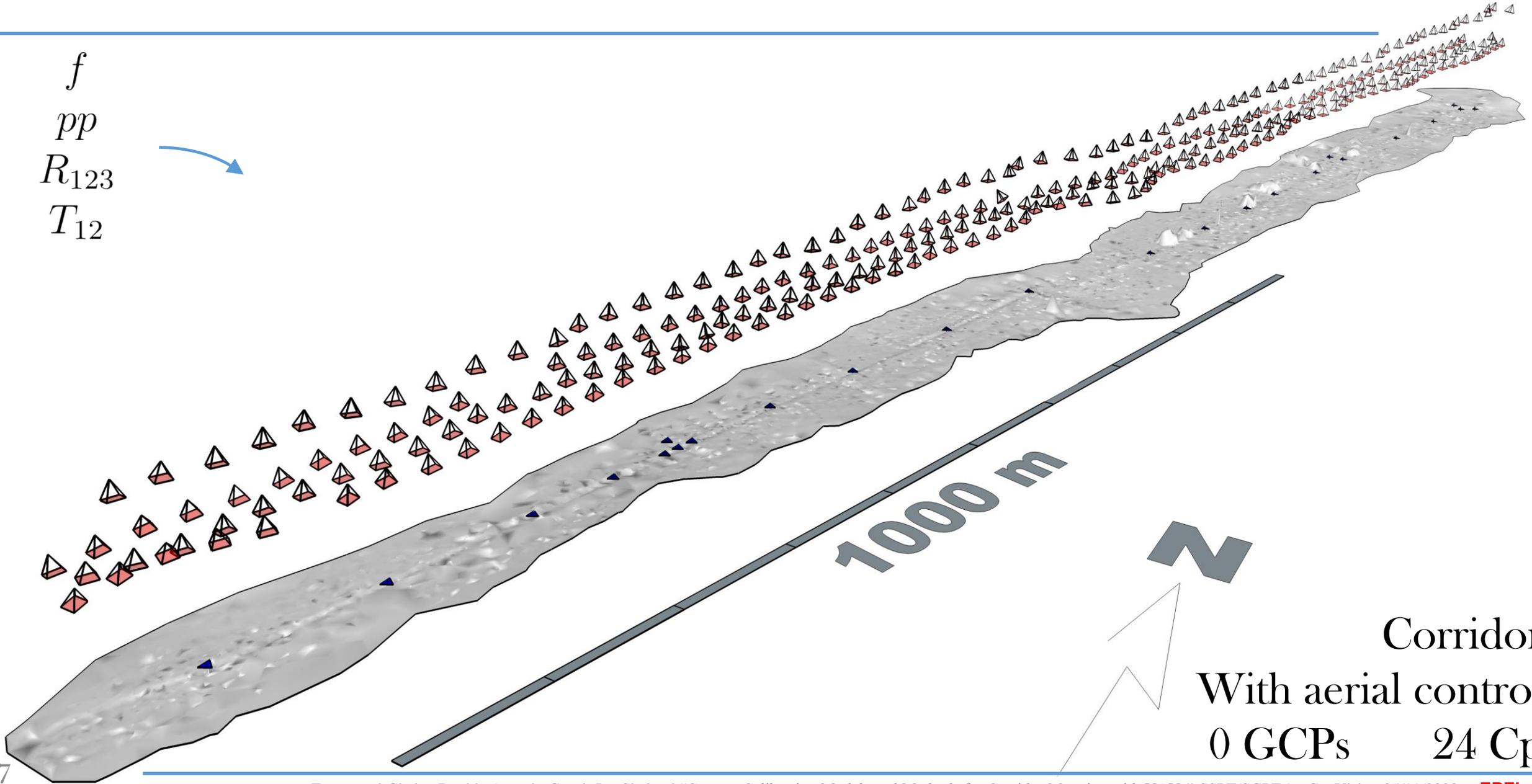


## ○ Usage mode for mapping

- FIX
- LEAD
- APC
- APCI

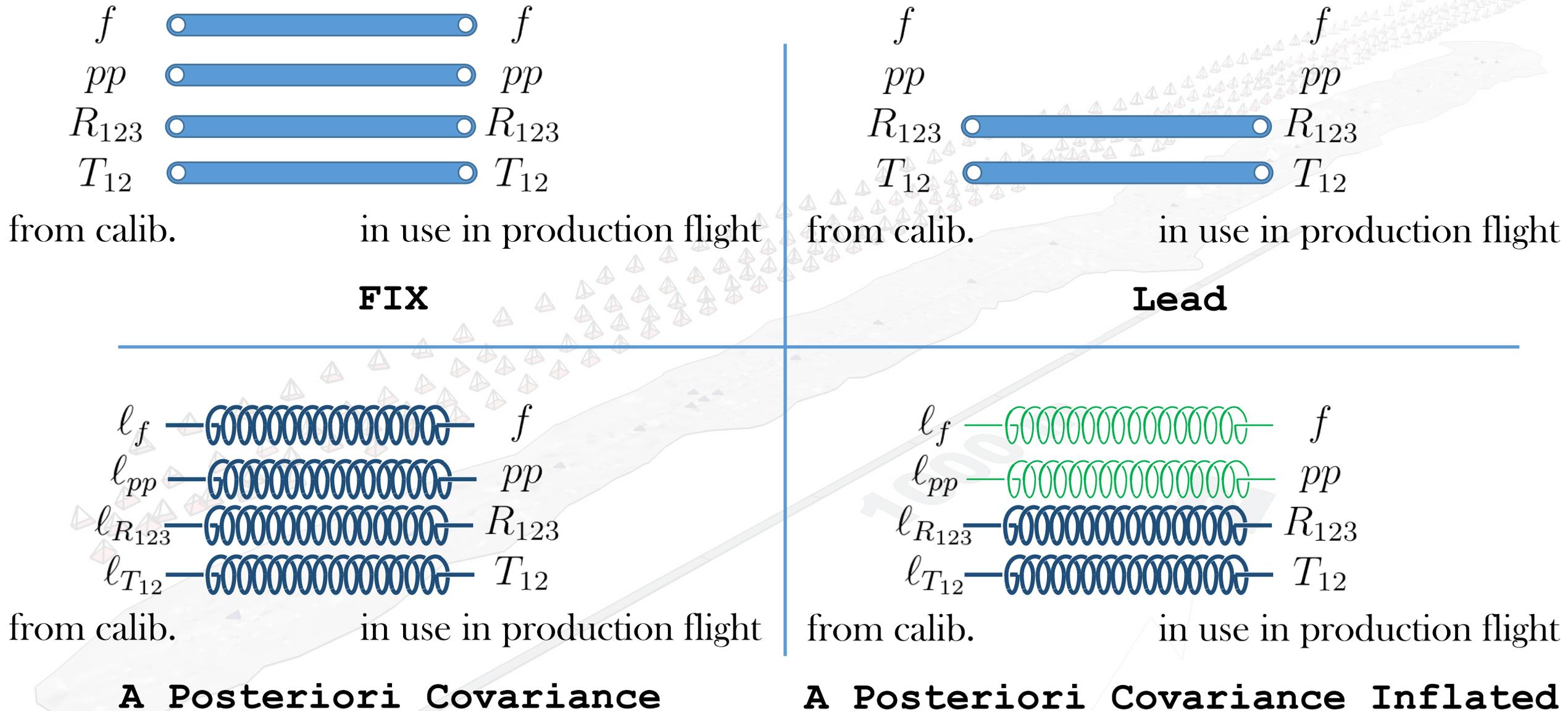
# Application: use of the calibration values

$f$   
 $pp$   
 $R_{123}$   
 $T_{12}$



Corridor  
With aerial control  
0 GCPs 24 Cp

# Application: use of the calibration values

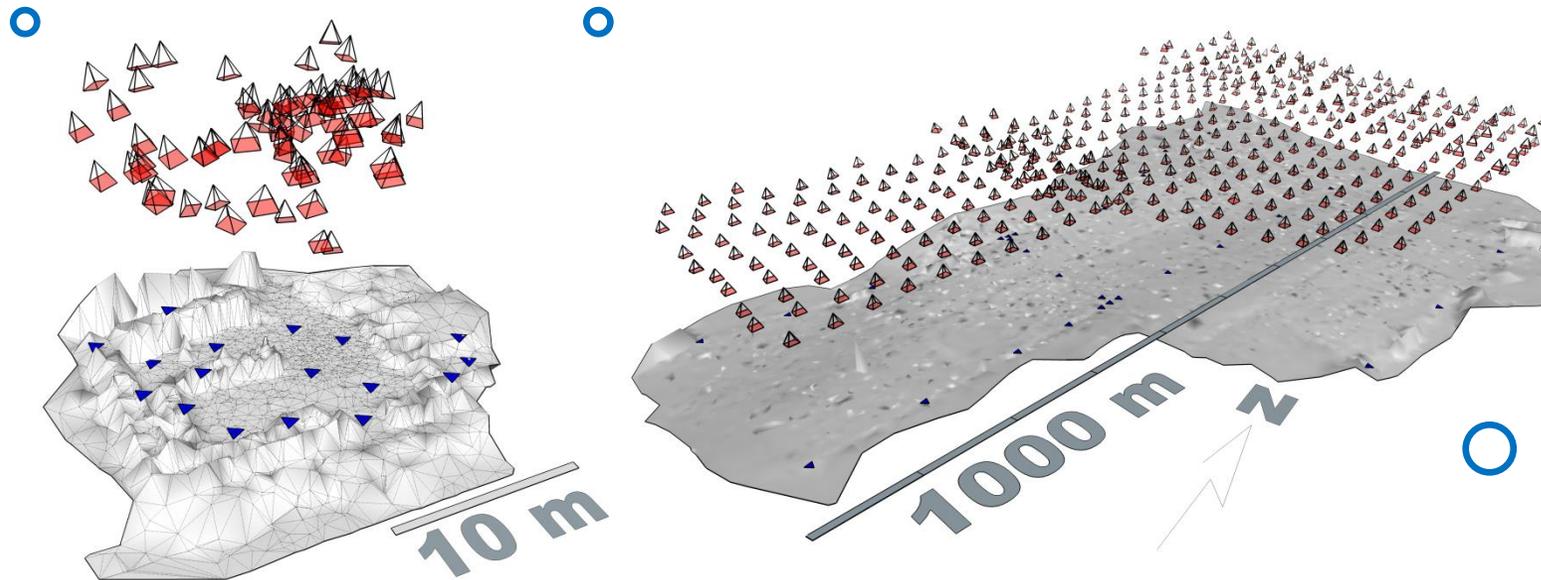


# Camera calibration model and method: evaluation in application

## ○ Camera model

- Brown 10
- Brown 15
- Brown 18
- Ortho-Poly 18
- Ortho-Poly 15

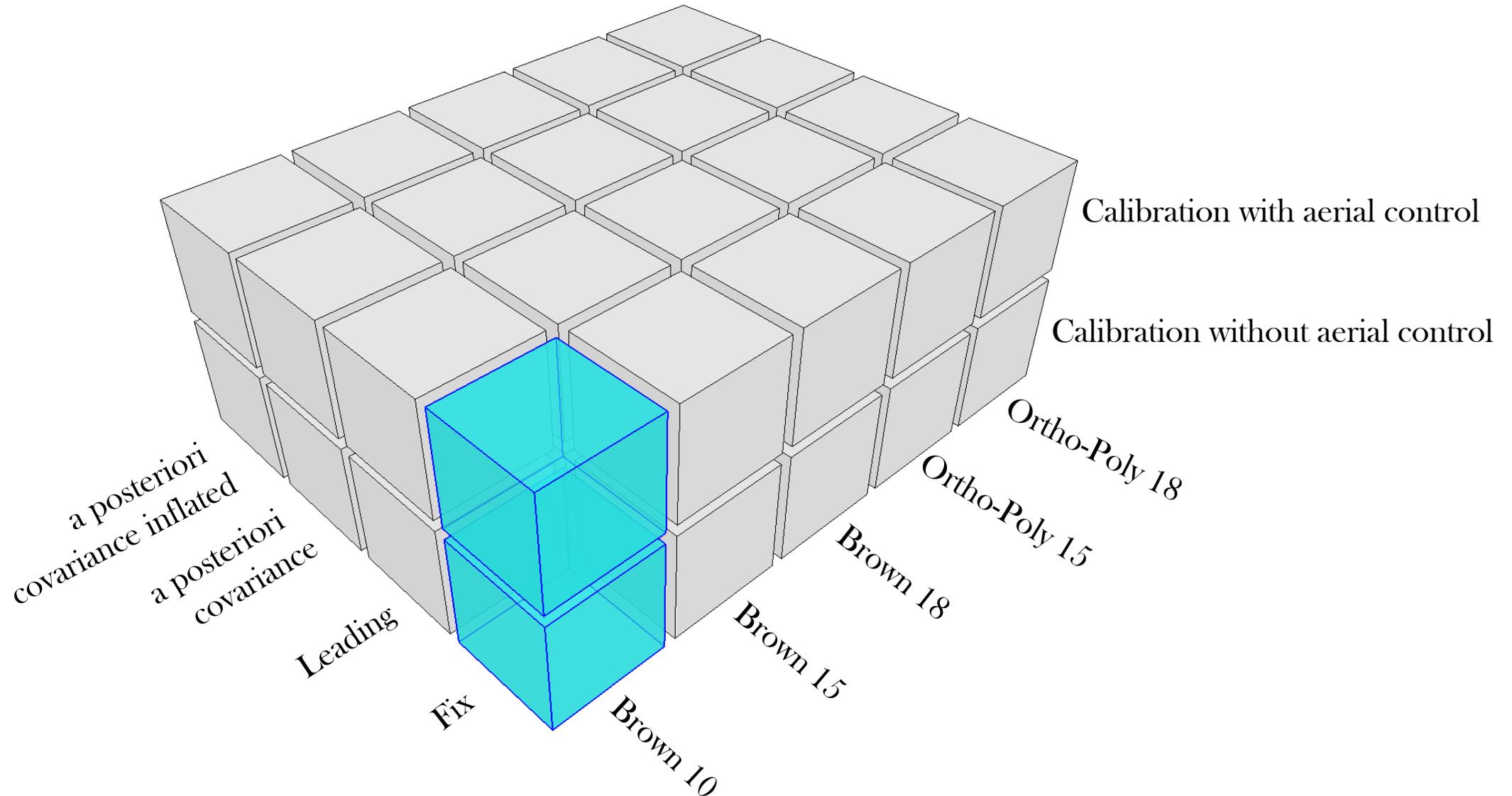
## ○ Calibration method



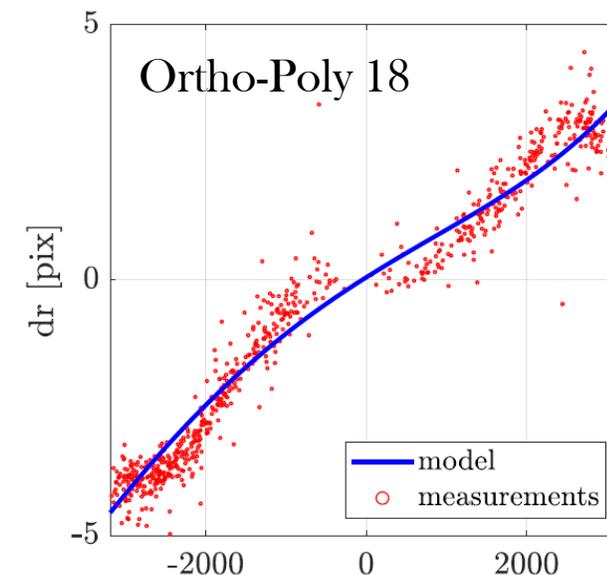
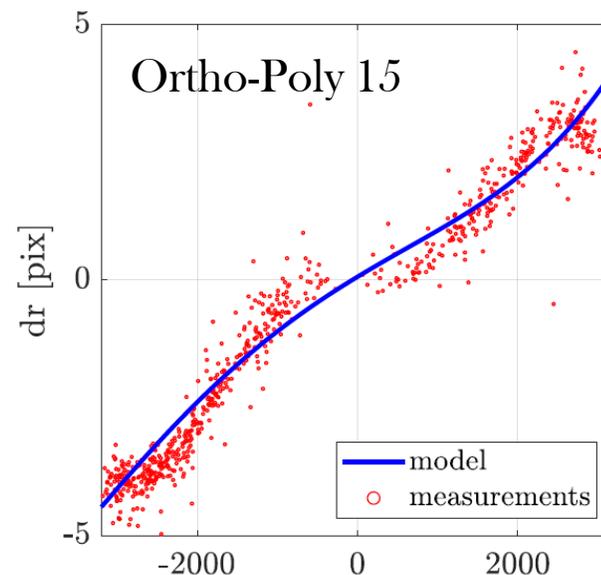
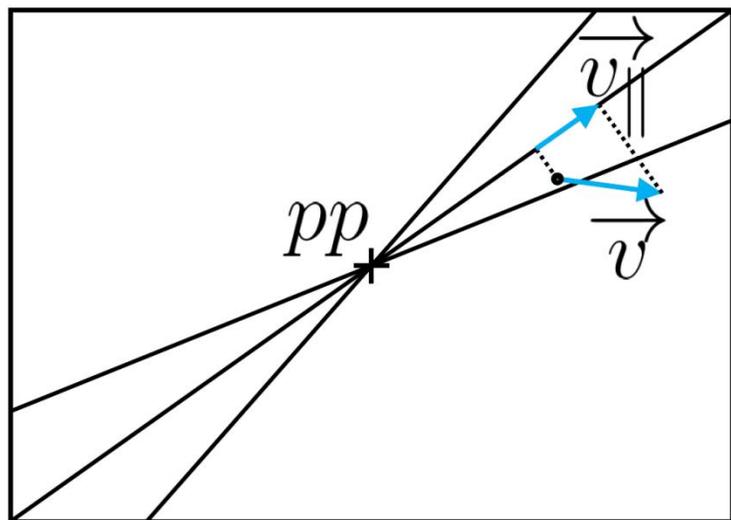
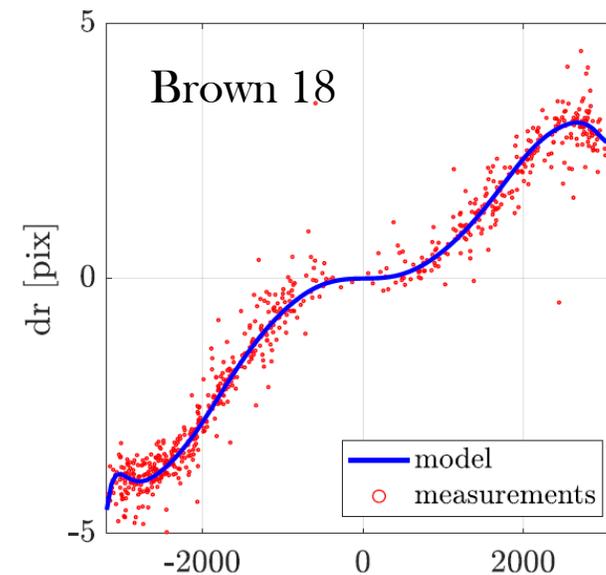
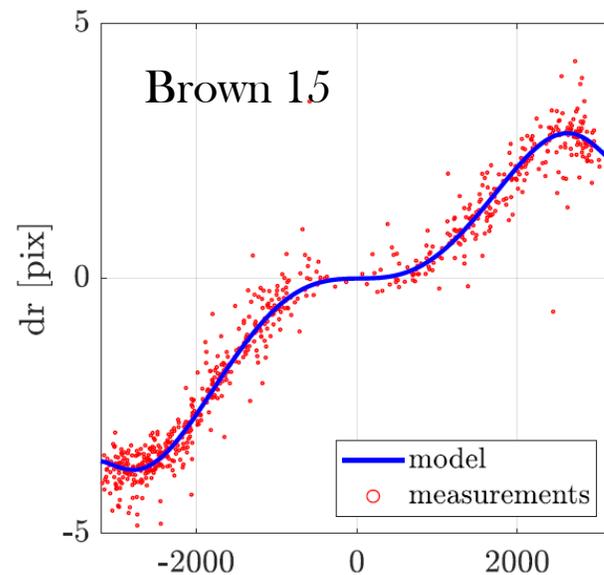
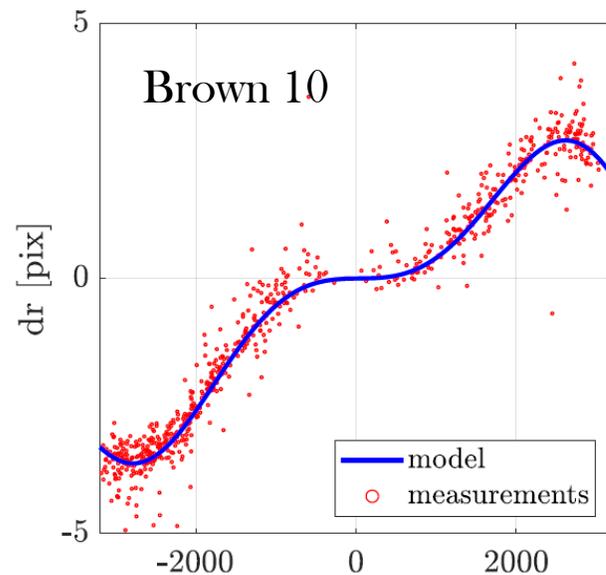
## ○ Usage mode for mapping

- FIX
- LEAD
- APC
- APCI

# Camera calibration model and method: Results & Conclusions



# Brown Vs Orthogonal Polynomials



# Camera calibration model and method: Results & Conclusions

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- **FIX** Fixing the camera calibration lead to inaccurate result (up to 3x worse than the other methods)
- **LEAD** Improves the results with respect to **FIX** and sometime yields to the best results
- **A Posteriori Covariance** and **A Posteriori Covariance Inflated** further improves the results
- Brown model gives results at least 2 times better than Orthogonal polynomials
- Better to perform a calibration flight in similar conditions as the mapping

# Questions?

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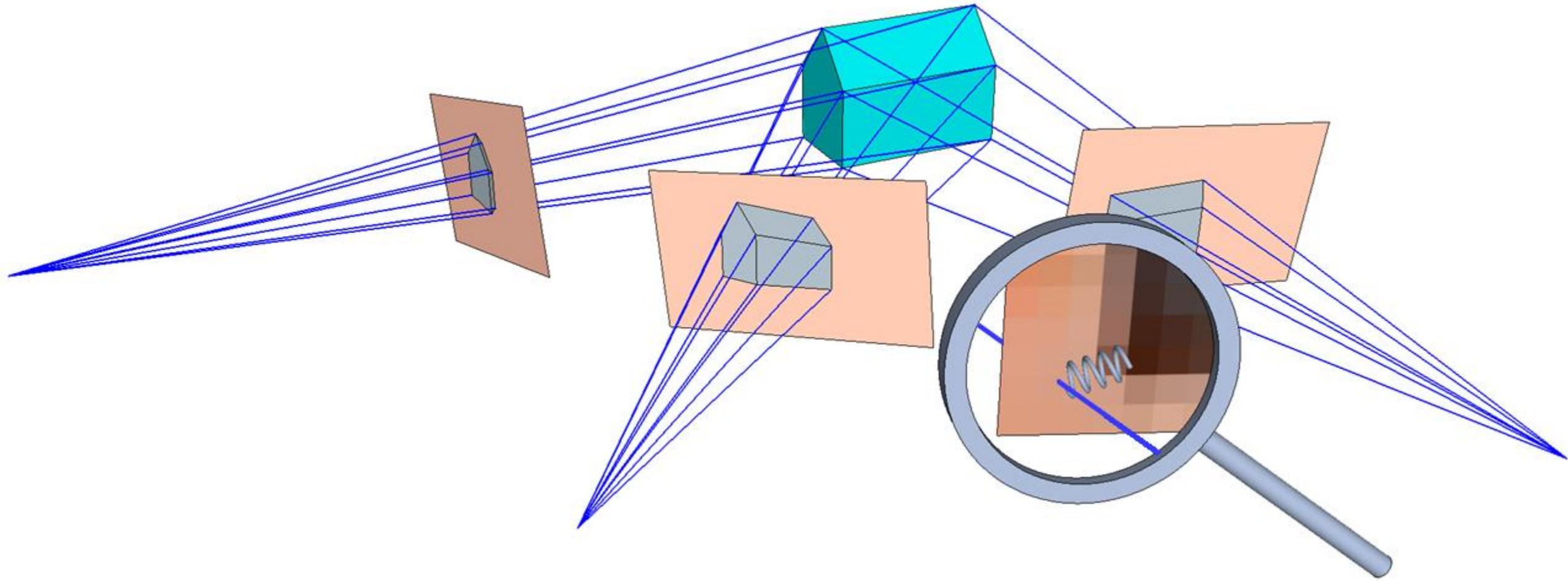
Clédats, E.; Cucci, D. A.; Skaloud, J. “Camera Calibration Models and Methods for Corridor Mapping with UAVs” *ISPRS Annals of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, XLIII 2020

# Backup slides

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# Bundle-Adjustment

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$$\xi_1: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$\tilde{p} = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} \mapsto (1 + K_1 \tilde{r}^2 + K_2 \tilde{r}^4 + K_3 \tilde{r}^6 + \dots) \tilde{p} + \begin{bmatrix} (P_1 (\tilde{r}^2 + 2\tilde{x}^2) + 2P_2 \tilde{x} \tilde{y}) \\ (2P_1 \tilde{x} \tilde{y} + P_2 (\tilde{r}^2 + 2\tilde{y}^2)) \end{bmatrix} (1 + P_3 \tilde{r}^2 + \dots)$$

where  $\tilde{r}^2 = \tilde{x}^2 + \tilde{y}^2$

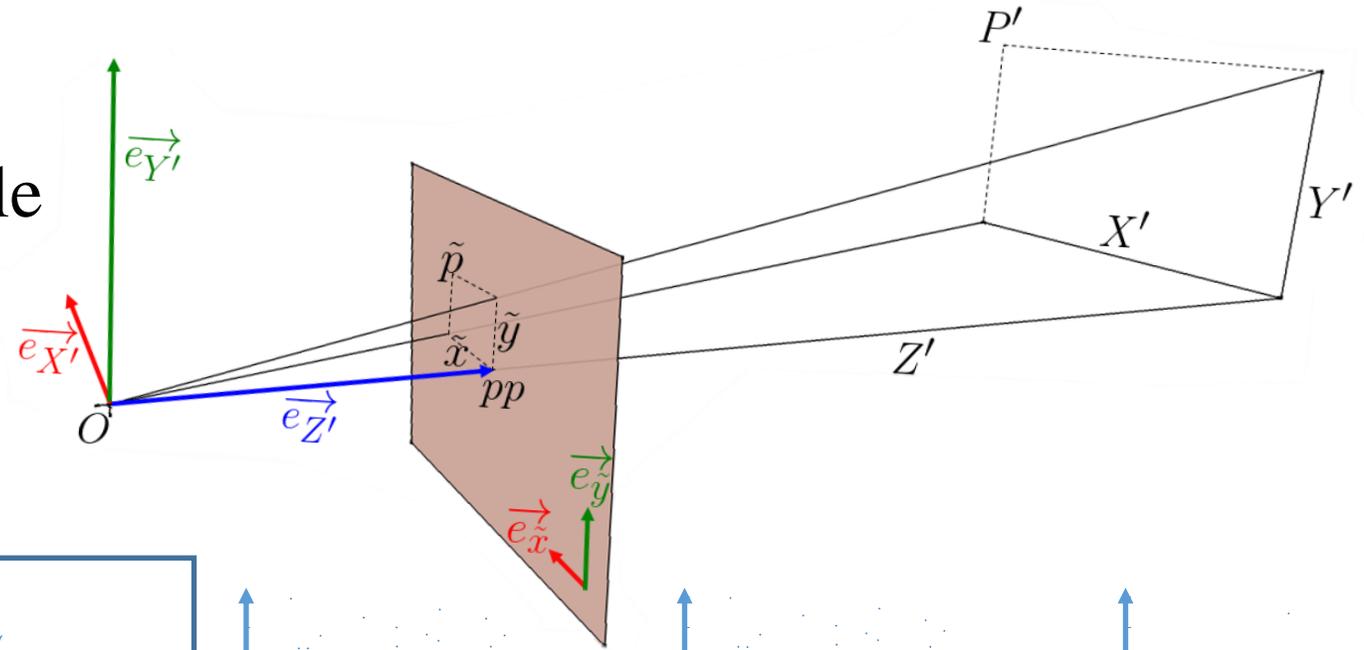
$$\xi_2: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} c + B_1 & B_2 \\ 0 & c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} pp_x \\ pp_y \end{bmatrix}$$

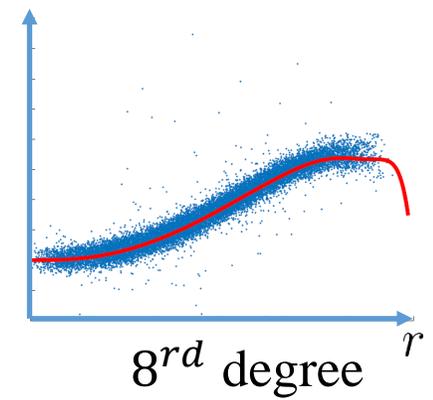
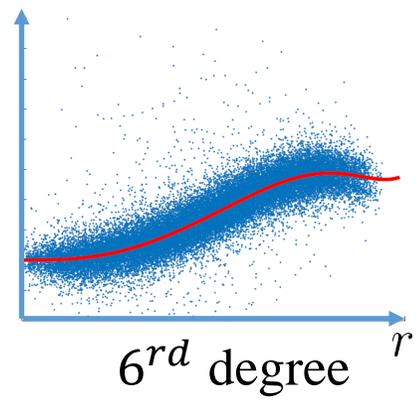
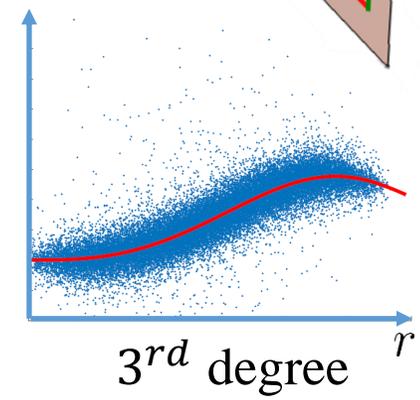
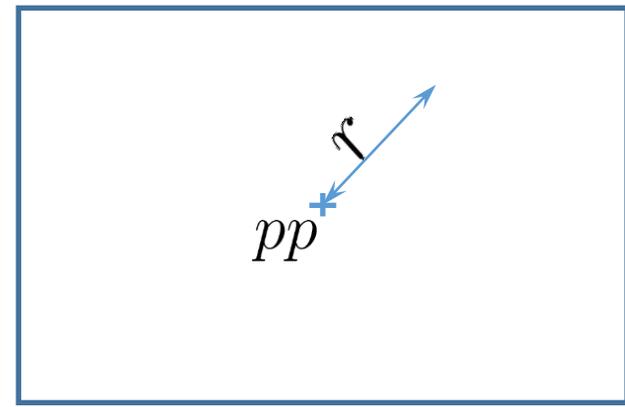
$$\xi = \xi_2 \circ \xi_1$$

# Camera models

- Departure from pinhole camera model



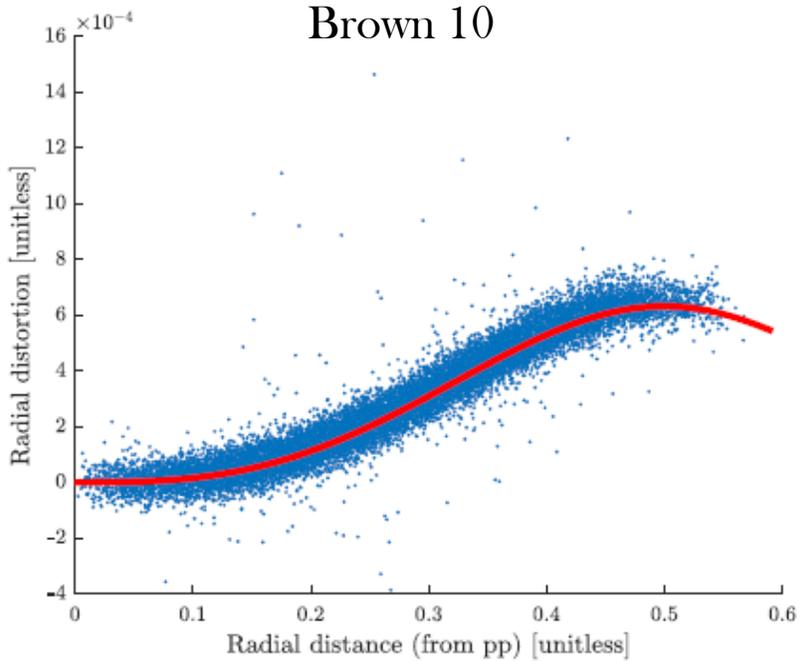
- Radial



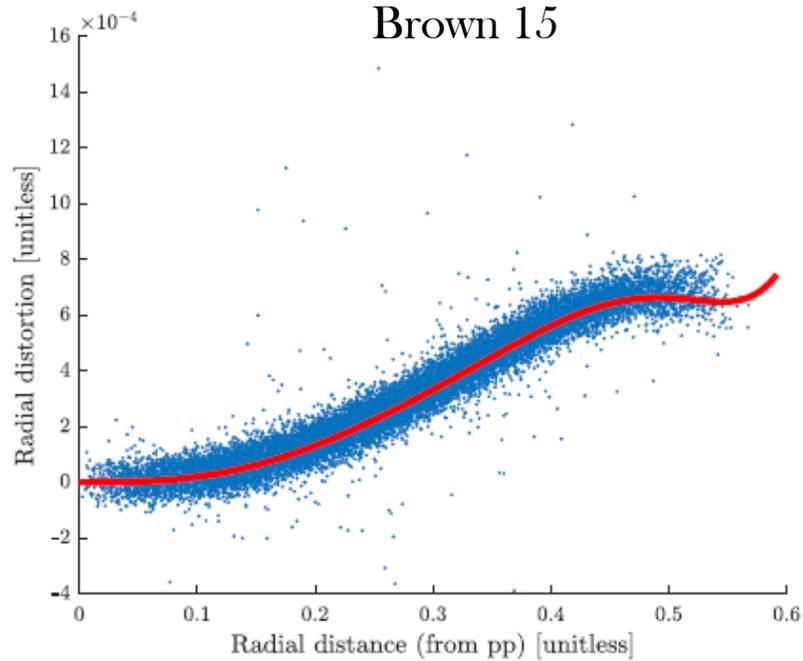
# Camera calibration



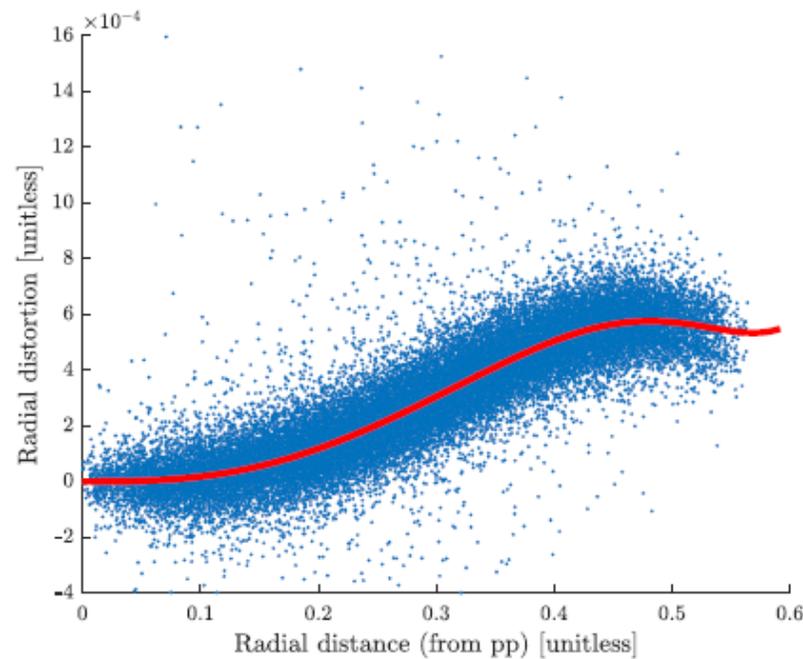
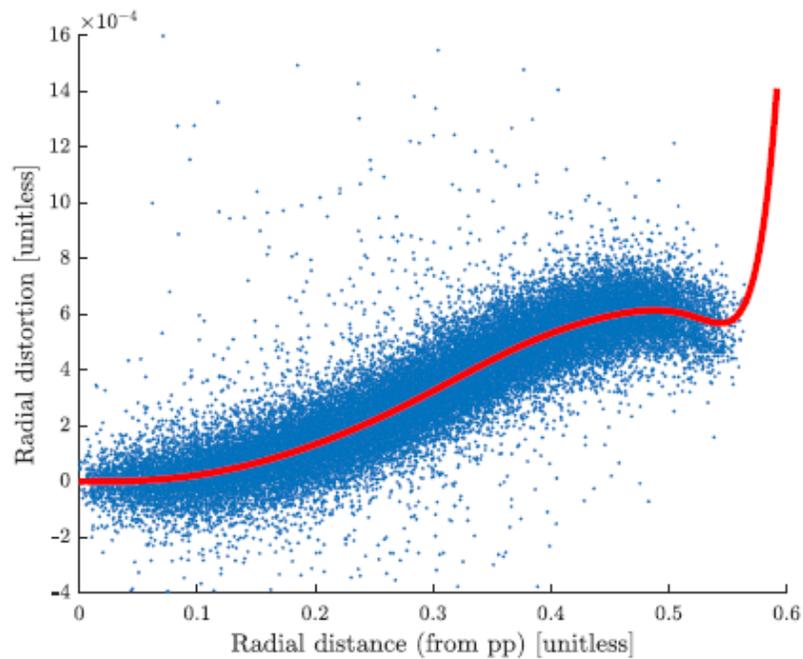
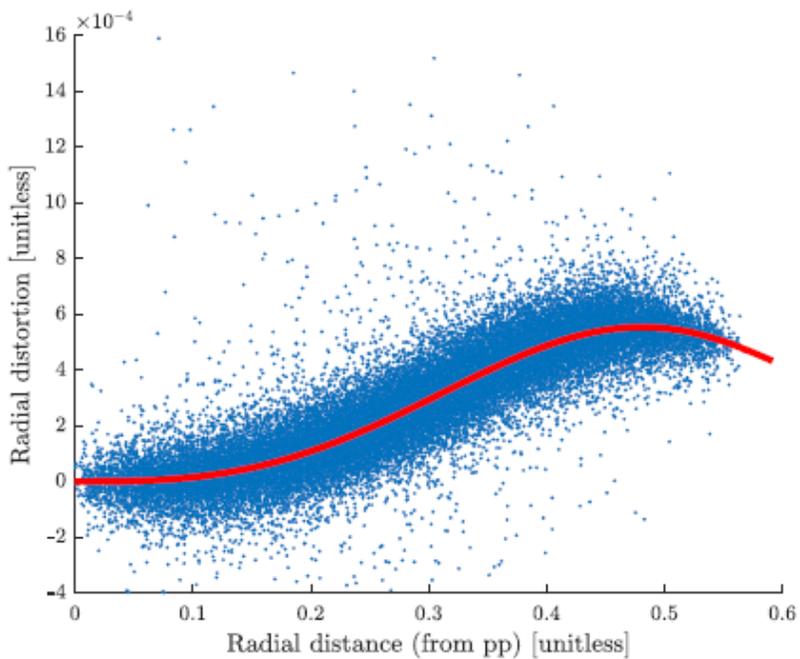
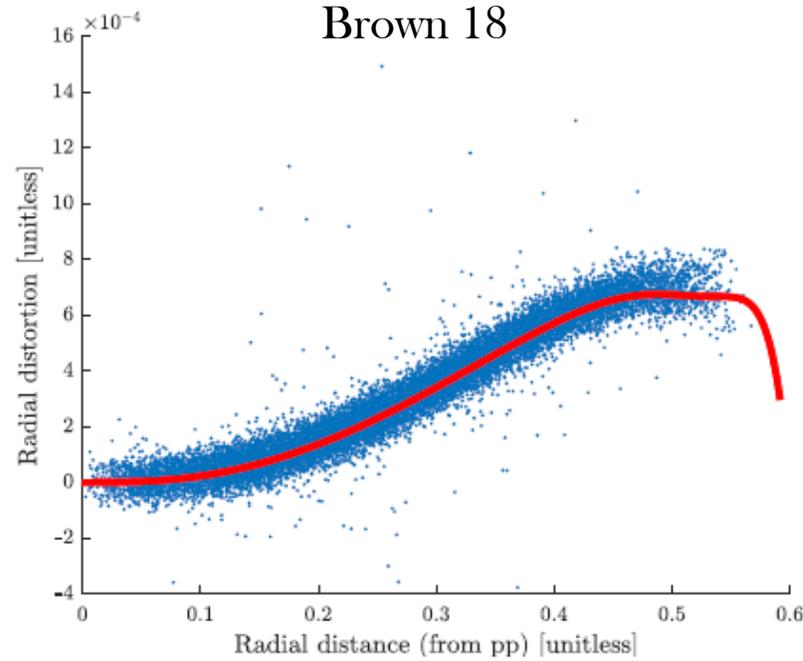
Brown 10



Brown 15



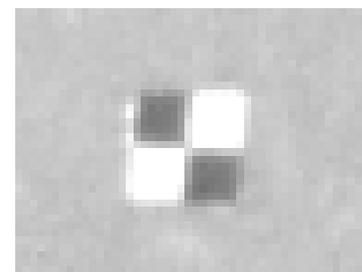
Brown 18



# Flight in test-field



200 m



30 cm